Digital Logic

- Boolean functions

AND(F, F) = F
AND(F, T) = F
AND(T, F) = F
AND(T, T) = T

$$f : \underbrace{\{0,1\}^n}_{\text{the set of all n-tuples}} \to \{0,1\}$$

TRUTH Table

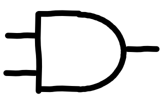
x	y	AND
0	0	0
0	1	0
1	0	0
1	1	1

variables

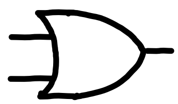
$x \cdot y$

x is a proposition
$x = T$ or $x = F$

- gates



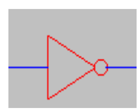
AND(x,y)



OR(x,y)

x	y	OR
0	0	0
0	1	1
1	0	1
1	1	1

$x + y$



NOT(x)

x	NOT
0	1
1	0

$\bar{x}$

- function Primitives (harmonic analysis)

function composition: $f(x) = s(g(x)) \Longrightarrow x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x))$

$$f(x,y) = r(\,g(x,y)\,,\,h(x,y)\,)$$

$$f(x,y) = \text{OR}(\ \text{AND}(\bullet,\bullet)\,,\ \text{AND}(\bullet,\bullet),\ \ldots\ \text{AND}(\bullet,\bullet)\ )$$

x or $\bar{x}$ y or $\bar{y}$

= min-term expansion of f

$$= \text{AND}(\ \text{OR}(\bullet,\bullet)\ \ \text{OR}(\bullet,\bullet),\ \ldots\ \text{OR}(\bullet,\bullet))$$

= max-term expansion of f

x	y	$f(x,y)$
0	0	0
0	1	1
1	0	1
1	1	0

=

x	y	$g(x,y)$
0	0	0
0	1	1
1	0	0
1	1	0

+
OR

x	y	$h(x,y)$
0	0	0
0	1	0
1	0	1
1	1	0

$$f(0,0) = g(0,0) + h(0,0)$$
$$f(0,1) = g(0,1) + h(0,1)$$
$$f(1,0) = g(1,0) + h(1,0)$$
$$f(1,1) = g(1,1) + h(1,1)$$

what is this function?

x	y	$g(x,y)$
0	0	0
0	1	1
1	0	0
1	1	0

AND(NOT(0) , 1) = 1

let's check, same as $g()$?

AND(NOT(0) , 0) = 0

AND(NOT(1) , 0) = 0

AND(NOT(1) , 1) = 0

$$g(x,y) = \text{AND}(\text{NOT}(x), y) = \bar{x} \cdot y$$

$$= \text{min-term } m_{01}(x,y) \quad \text{aka} \quad m_1(x,y)$$

x	y	$h(x,y)$
0	0	0
0	1	0
1	0	1
1	1	0

$\rightarrow x \cdot \bar{y}$ or m_2

the others

$m_0 = \bar{x} \cdot \bar{y}$

$m_3 = x \cdot y$

$$\Rightarrow f(x,y) = g(x,y) + h(x,y)$$

$$= \bar{x} \cdot y + x \cdot \bar{y} = m_1 + m_2$$

in general

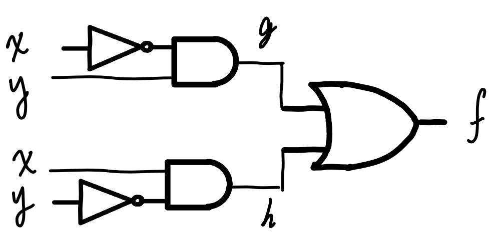
$$\left(f(\cdot) = \sum_{0}^{2^n - 1} a_i \cdot m_i \right)$$

OR

$a_i \in \{0,1\}$

AND

$\Rightarrow$



This "minterm expansion" method can be extended to boolean functions of any number of arguments (*n*-ary functions):

> **Any *n*-ary logic function can be built as an OR of minterms.**

Minterms are n-input ANDs of variables or their negations.