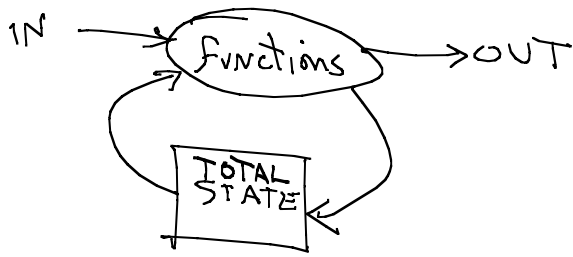


FSM implementation



- need to build
1. Symbols
 2. functions
 3. State
 - (4. tape, for UTM/computer)

Any technology will do!

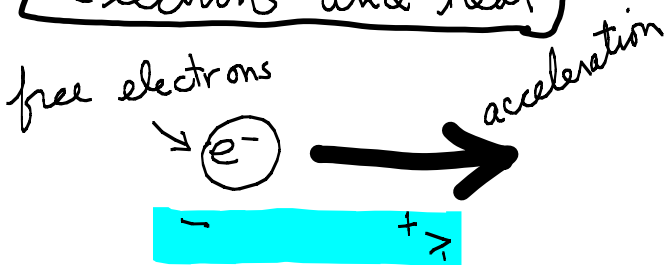
mechanical: Babbage, etc.

electro-mechanical ← we start here

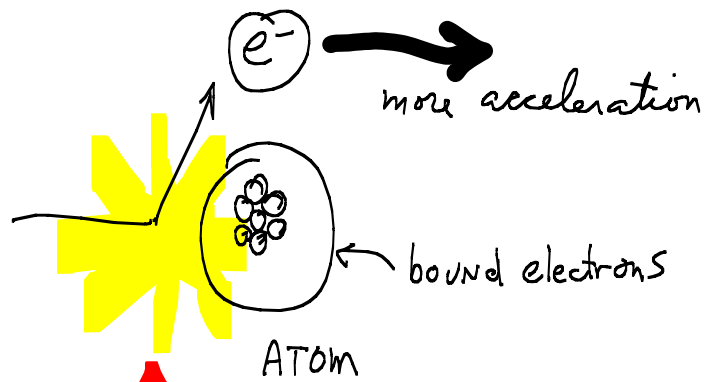
electronic

?

Electrons and heat

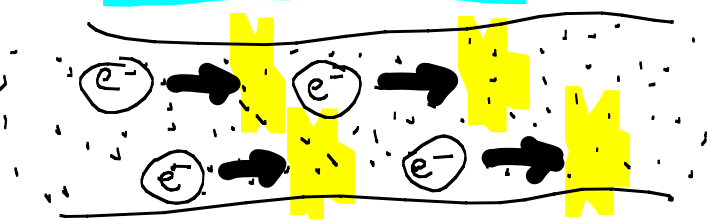


$E_{lec. field} \approx gravity$



$$E \times distance = Voltage$$

Voltage



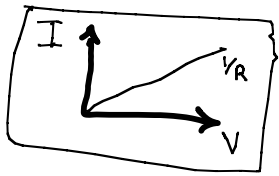
High current = Lots of moving e^-
 High voltage = Lots of acceleration

High Voltage + High current = Lots of Heat = $\frac{energy}{sec} = Power = I \cdot V$

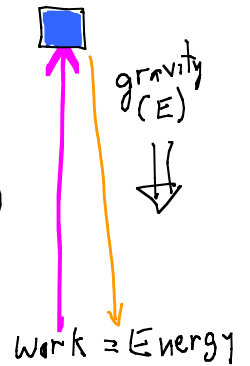
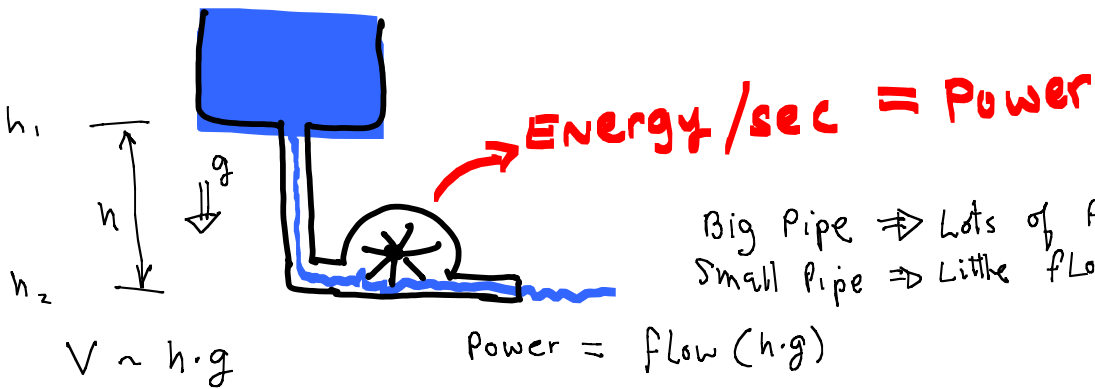
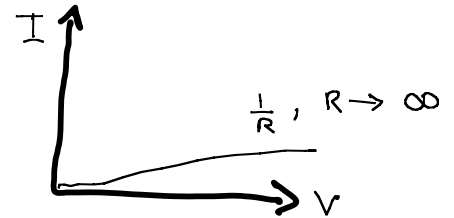
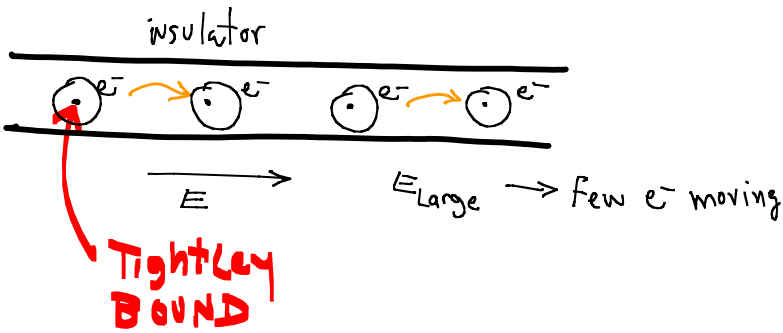
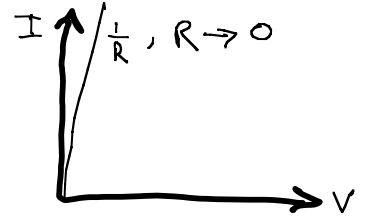
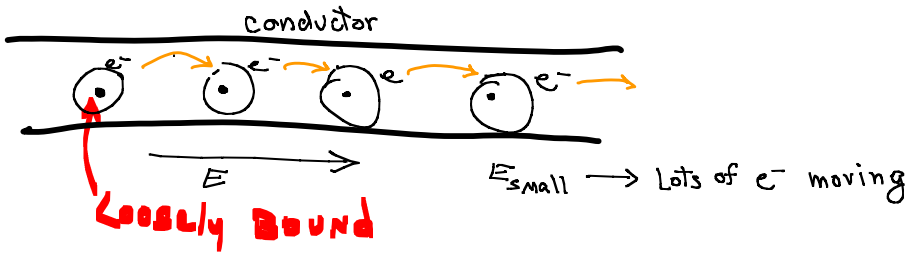
Collision transfers energy to atoms of material, e.g. wire.
 Atom motion = heat.
 ~ All collision energy goes to heat.

Ohm's Law, conductors, ...

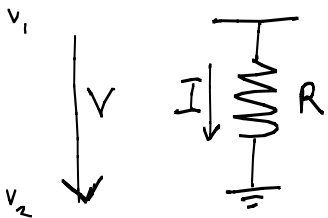
$R \stackrel{\text{def.}}{=} V/I$



$\Rightarrow I = (1/R)V$ $\Rightarrow V = IR$
 (if relationship is linear, then Ohm's Law Resistor) OR Linear device.



$V = E \times \text{distance}$
 \sim Tower height \times gravity



$V = IR$

fix V: as $R \rightarrow 0$ then $I \rightarrow \infty$
 as $R \rightarrow \infty$ then $I \rightarrow 0$

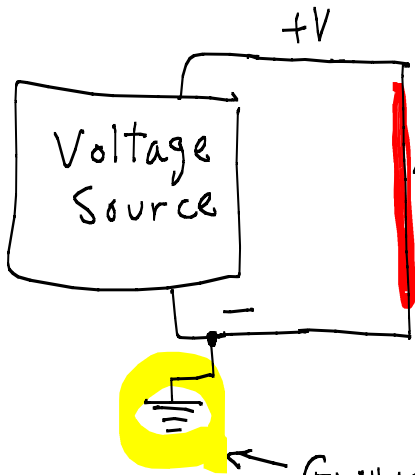
$V = V_1 - V_2$

Power = $I V$
 ← Energy over distance per e^-
 ← $\times e^-$ per second

power = energy/sec
 $= I \cdot V = I \cdot (I \cdot R) = I^2 \cdot R$
 $= I \cdot V = (V/R) \cdot V$

$R \rightarrow 0$ then Power $\rightarrow \infty$
 $R \rightarrow \infty$ then Power $\rightarrow 0$





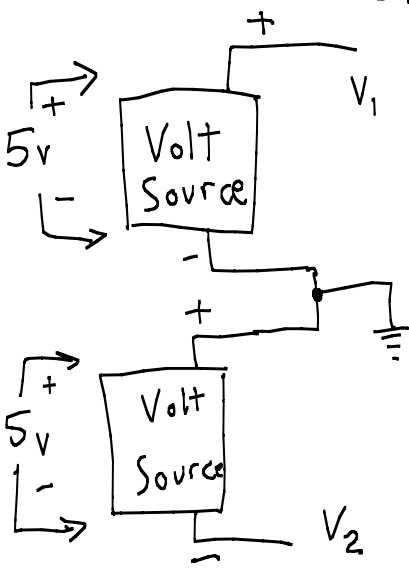
What if this is wire, ie., R is nearly 0?
Voltage source holds constant V voltage difference.

Then current $I = V/R$ goes to infinity.

Then power dissipated to heat = $I*V$ goes to infinity.

Infinite release of ENERGY! BOOM! (Actually, wire and/or supply melts, Ohm's Law not a good approximation at that point.)

Ground symbol $\equiv 0$ volts, by definition

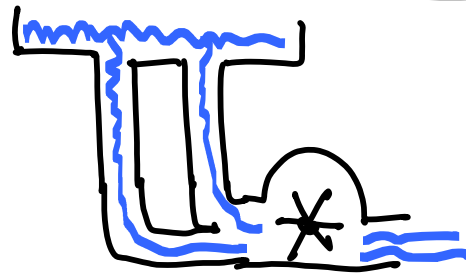
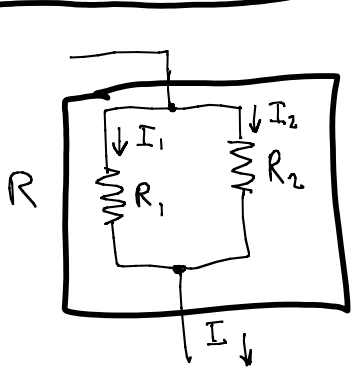


Measure voltage difference from voltage point at GROUND.

What voltage do we call V1?

What voltage do we call V2?

V1 = +5v relative to GND
V2 = -5v relative to GND



Resistors connected in PARALLEL.

Same as two water pipes in parallel: less resistance to flow, total flow is sum of flow in both.

Water pressure, voltage, is same for both paths.

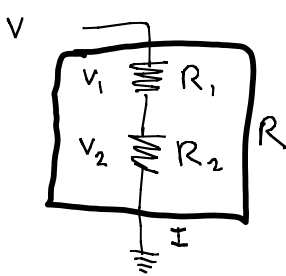
$i = i_1 + i_2$

If $R_1 = R_2$ Then $i_1 = V/R_1 = i_2$

$i = i_1 + i_2 = 2*V/R_1$

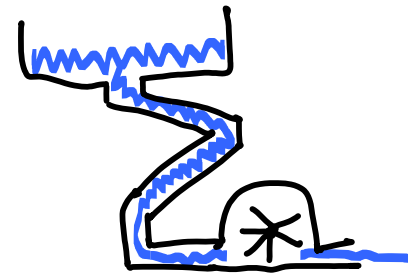
What is total R?

$R = V/i = V/(2V/R_1) = 1/2 R_1$



Resistors connected in SERIES.

Same as two pipes end-to-end: more resistance, less flow, same flow in both.



$R = V/i = (V_1 + V_2)/i$

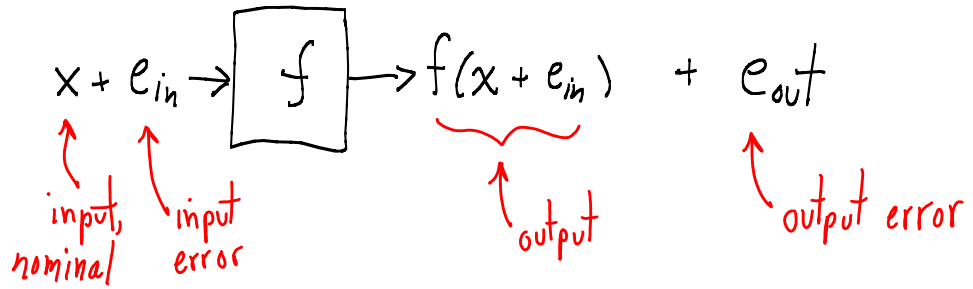
$V_1 = i*R_1 \quad V_2 = i*R_2$

$R = (i*R_1 + i*R_2)/i = (R_1 + R_2)$

We need Signal-Restoring, Non-Linear Logic. Ohm's Law devices are LINEAR.

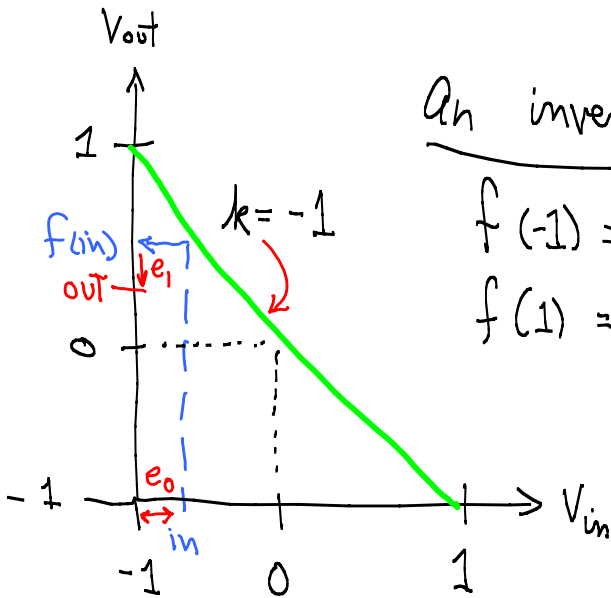
Suppose we had only linear devices (or something very nearly linear), then signal output has errors proportional to input errors.

Errors/noise :

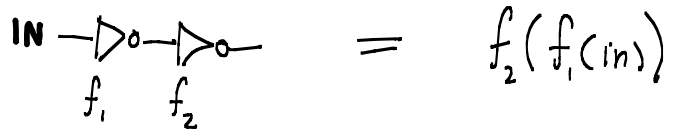


Linearity:

$$f(v + e_{in}) = k(v + e_{in}) + e_{out} = kv + \underbrace{ke_{in}}_{\text{Total error}} + e_{out}$$



Suppose we connect 2 in series



$$\begin{aligned}
 f_1(in) &= f_1(v + e) = kv + ke_0 + e_1 \\
 &= (-1)(-1) + (-1)e_0 + e_1 \\
 &= 1 - e_0 + e_1
 \end{aligned}$$

(v is nominal signal: ± 1)

$$\begin{aligned}
 f_2(f_1(in)) &= f_2(1 - e_0 + e_1) \\
 &= -1 + e_0 - e_1 + e_2
 \end{aligned}$$

k stages $\Rightarrow 1 + \sum_{i=0}^k e_i (-1)^i$

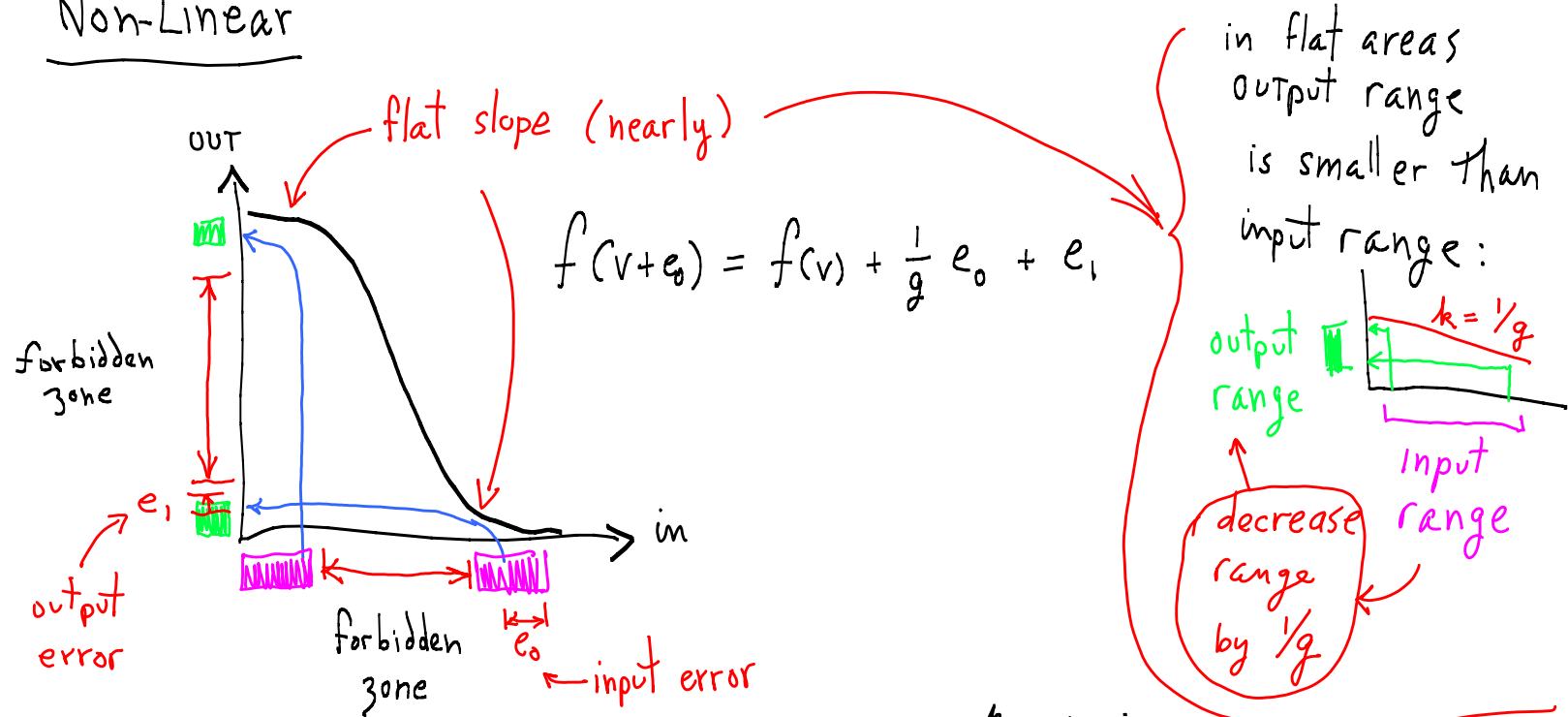
The errors include signs = random walk with random size steps.

Errors independently random w/ average 0 \Rightarrow variance increases w/ k.
 Total error grows w/o bound!

Take random step (either in the -1 or +1 direction).
 How far from 0 can you expect to be after k steps? About $k^{1/2}$ away.
 With probability 0 you will be at 0, and error gets unboundedly large.

We must Reduce error at each stage ==> exponentially decreasing effect in later stages.

Non-Linear



after k stages:
$$= 1 + \sum_{i=0}^k \left(\frac{1}{g}\right)^i e_i$$

Converges!

If g is large enough (flat areas of curve are flat enough),

and

if output error size is not too big,

Then

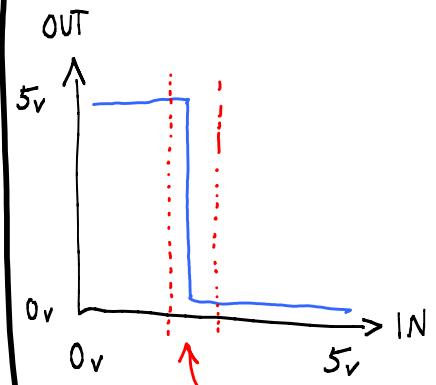
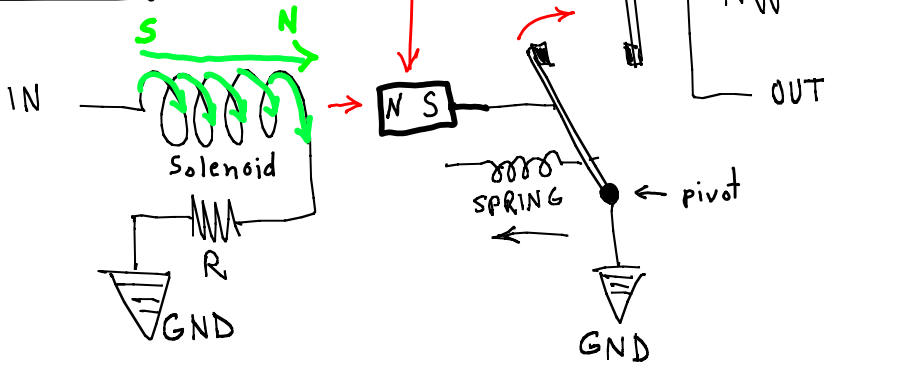
output after k stages never hits FORBIDDEN ZONE.

So, if we plan to have a circuit with long device chains, we must have non-linear devices w/ suitable response curves.

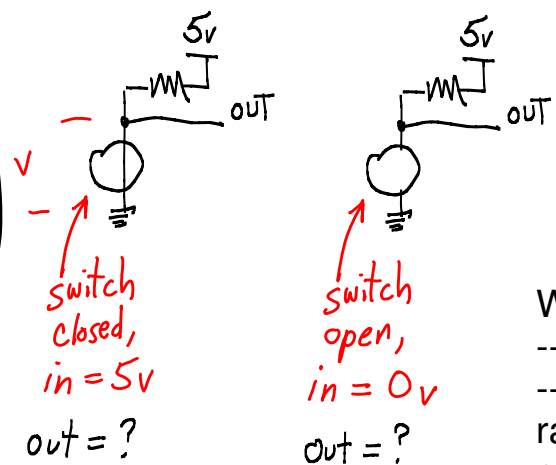
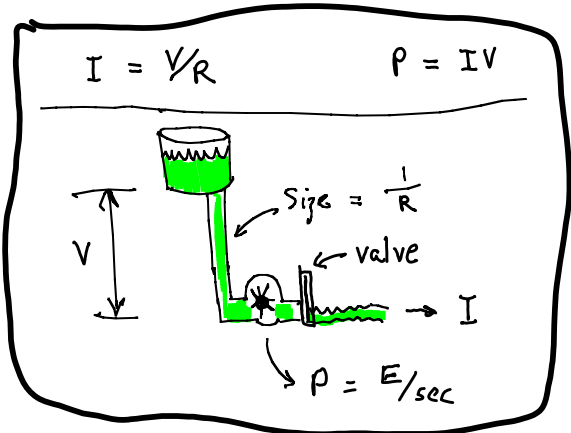
Do we plan to have long chains? YES:

- (1) feedback in system,
- (2) chained data operations: $D1 \implies D2 \implies D3 \implies D4 \dots$
- (3) 1 Billion devices per cpu

Devices



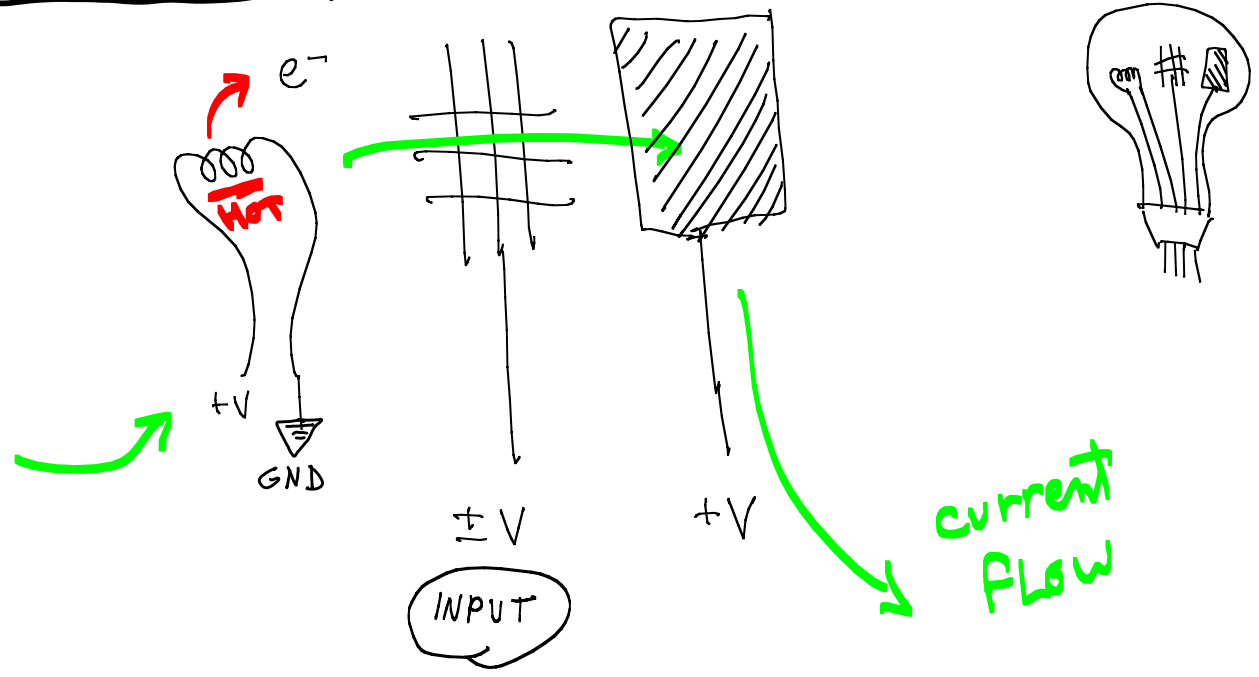
$V = IR$



switch closes, varies for each device
 ⇒ FORBIDDEN ZONE

What is the function?
 --- non-linear response
 --- any voltage in allowed range will yield "clean" output. Easy to prevent input voltages in forbidden zone.

Electronic Switches: Vacuum Tubes

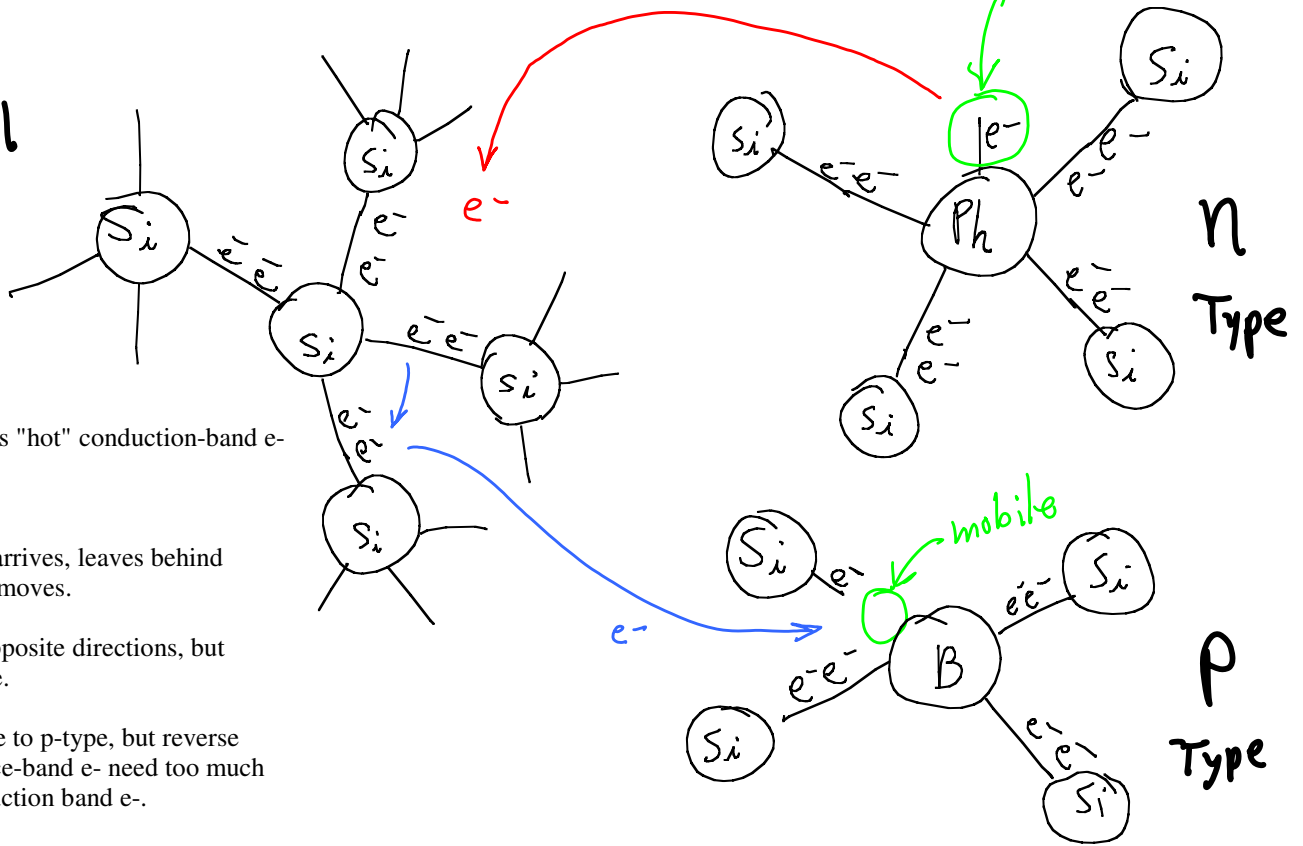


Faster, more reliable, less power

⇒ Voltage Controlled Switch

Solid state devices - Semiconductors

Silicon Crystal



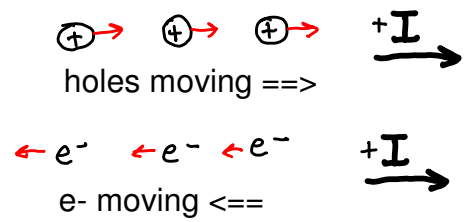
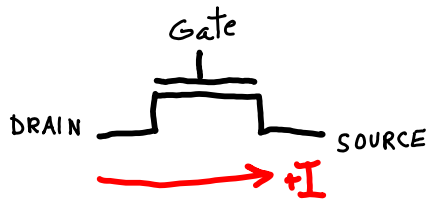
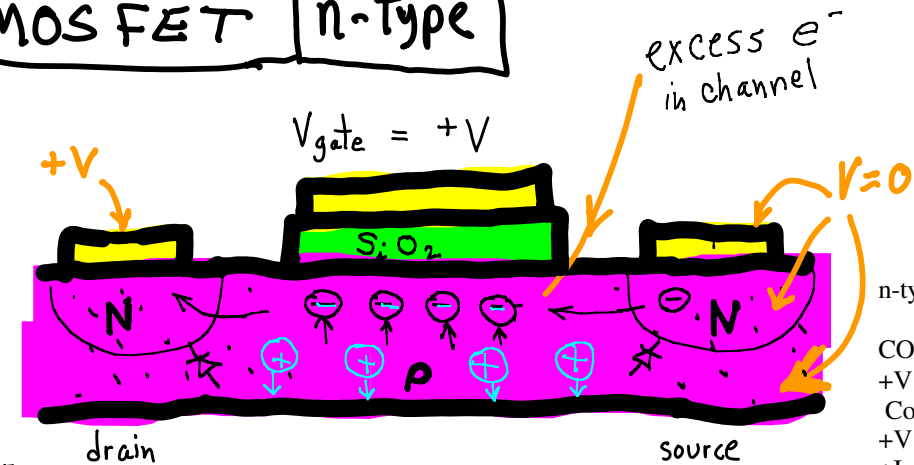
Phosphorous impurity:
 e^- leaves easily, becomes "hot" conduction-band e^- .

Boron impurity:
 "cold" valence-band e^- arrives, leaves behind +valence "hole" which moves.

Holes and e^- move in opposite directions, but current direction is same.

Easy e^- flow from n-type to p-type, but reverse flow hard: "cold" valence-band e^- need too much energy to become conduction band e^- .

MOSFET n-type



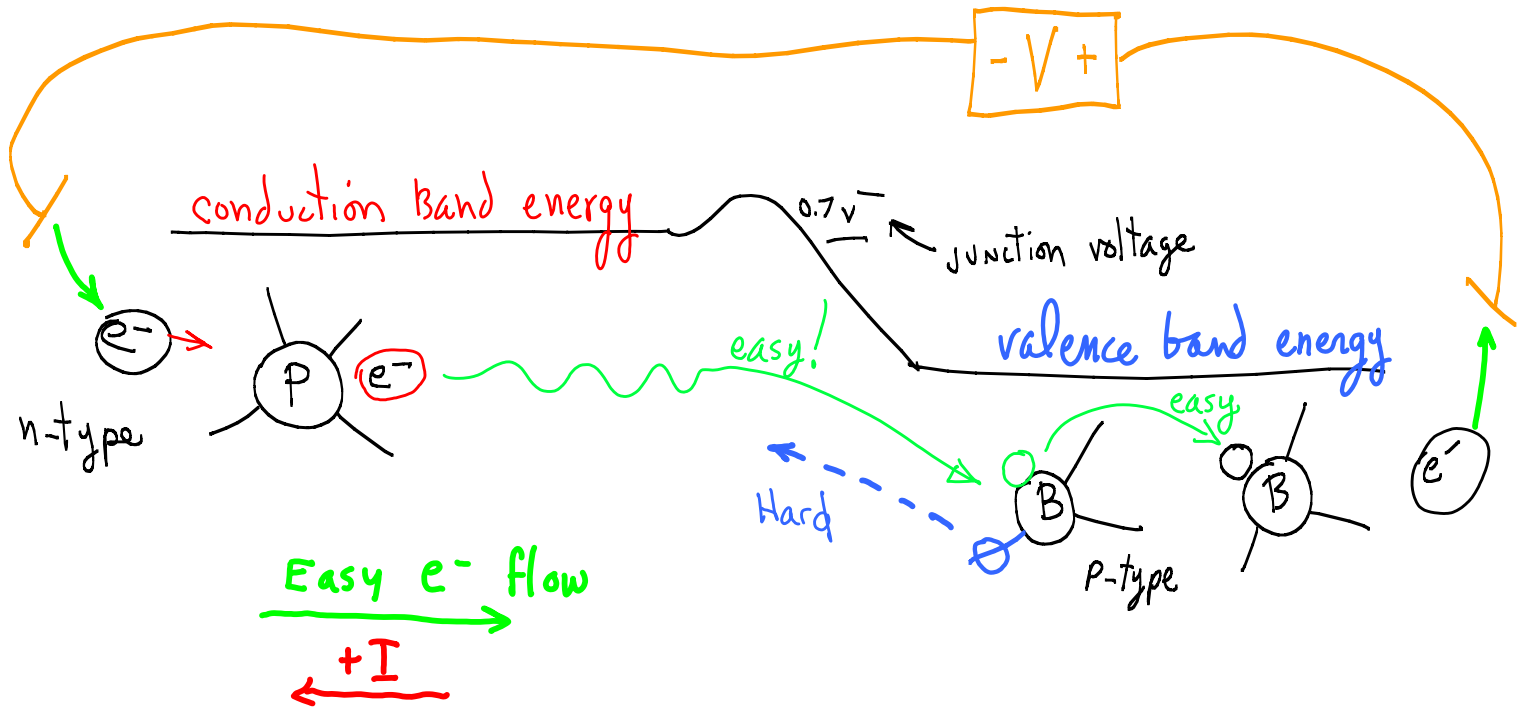
n-type MOSFET (n-transistor)

CONDUCTING ($V_{gate} = +V$, $R_{drain-source} \approx 0$):
 $+V$ on gate drives holes away from P-type channel.
 Conduction-band e^- move from source N-type well.
 $+V$ on drain pulls conduction-band e^- off.
 $+I$ current flows left-to-right.

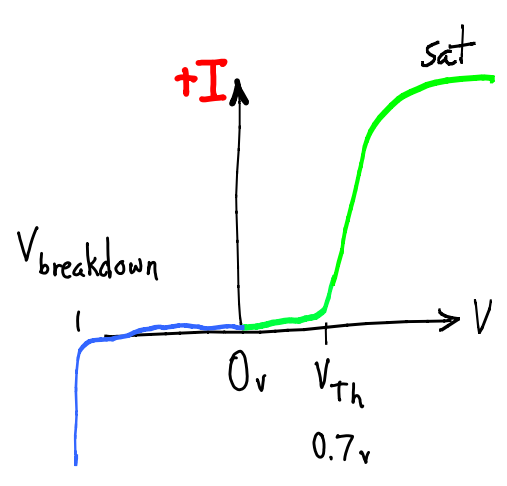
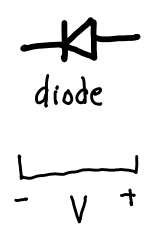
NOT-CONDUCTING ($V_{gate} = 0$, $R_{drain-source} = \text{BIG}$):
 $V_{gate} = 0$, holes populate channel.
 Source N-well e^- drop into valence band in channel.
 $+V$ at drain cannot pull valence-band e^- from P-type to N-type.

n-type transistor

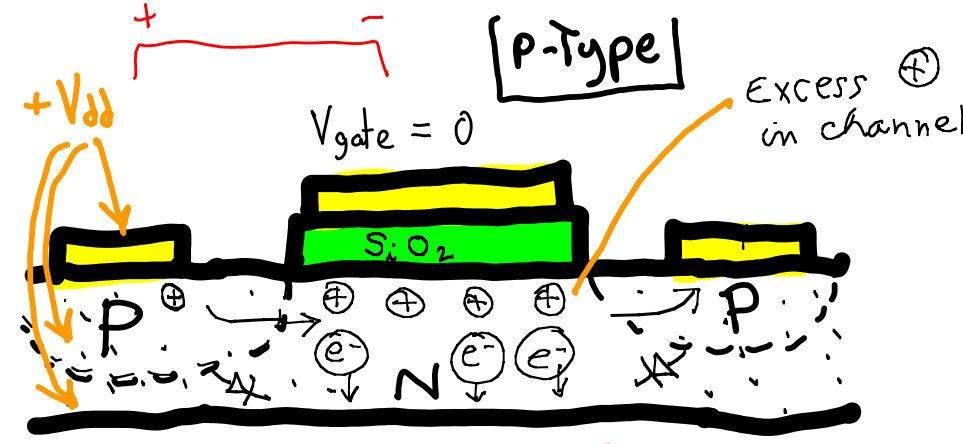
V_{gate}	$R_{drain-source}$
0	∞ not conducting
+V	0 conducting



Easy e^- flow
 $+I$



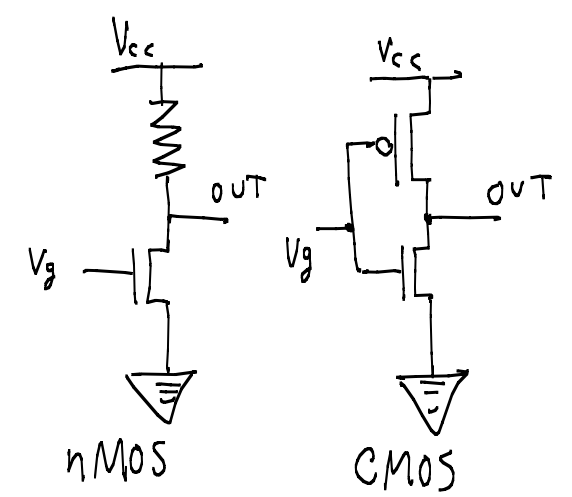
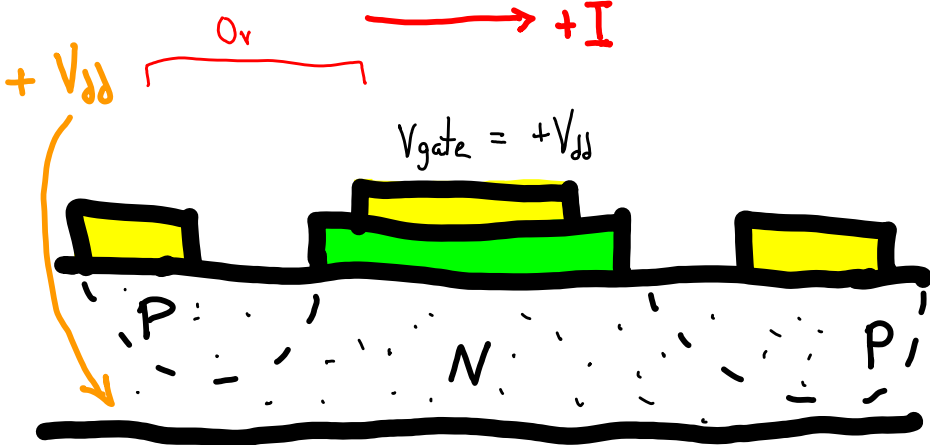
Just what we want: nice non-linear switch.



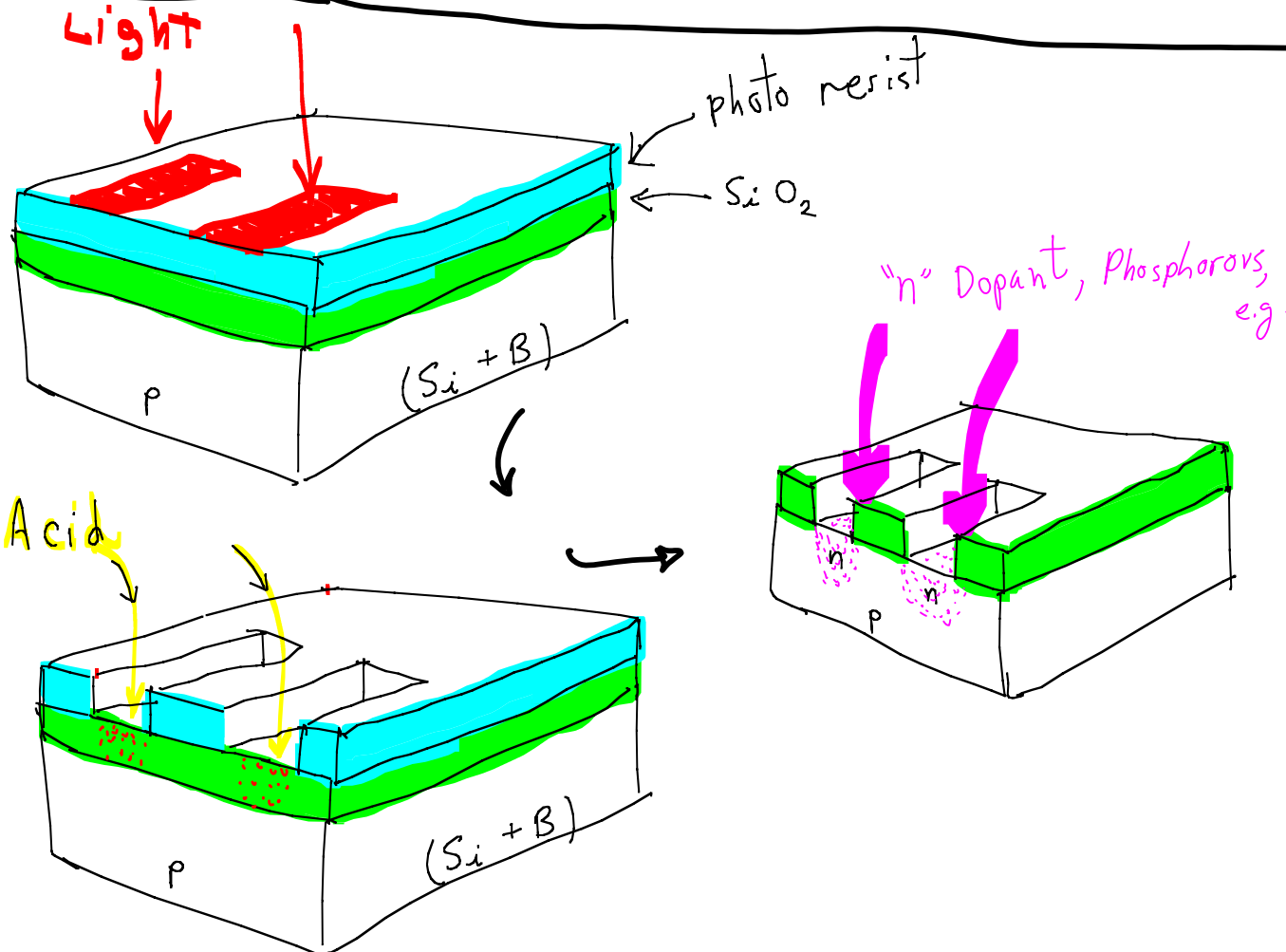
P-type (n-channel) transistor

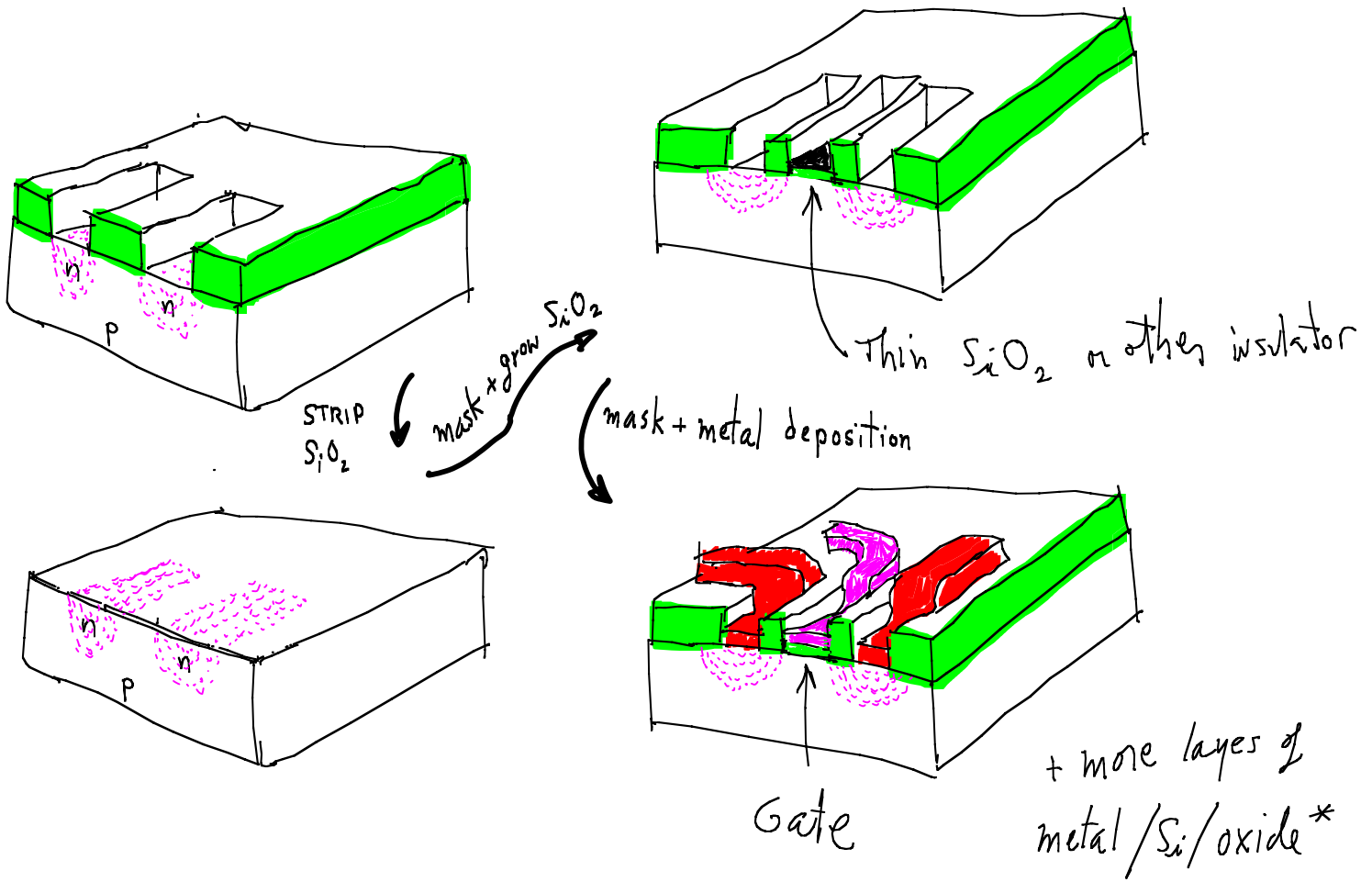
$V_{gate} = 0$:
 $V_{gate} = -V_{dd}$ wrt to base,
 pushes e^- away from channel, leaves
 excess holes, current flows.
 $R = 0$, conducting.

$V_{gate} = V_{dd}$:
 $V_{gate} = 0$ wrt to base
 channel is neutral
 only random thermal e^- available for
 current flow.
 $R = \text{infinity}$, not conducting

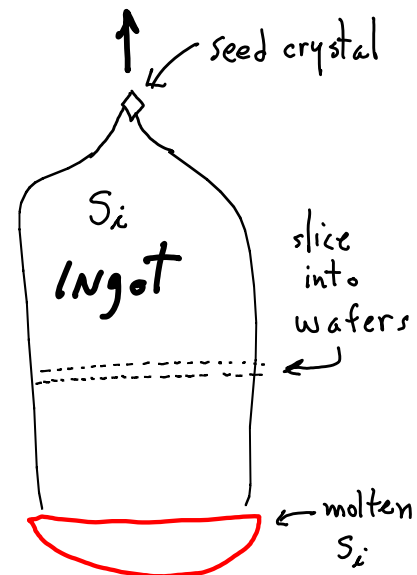
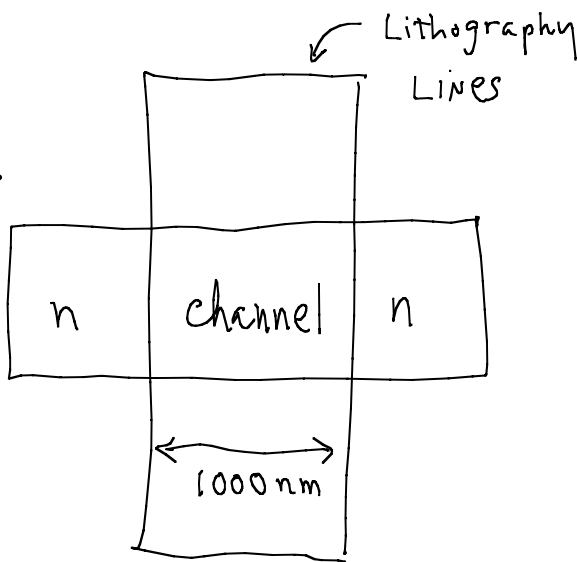


chip fab.

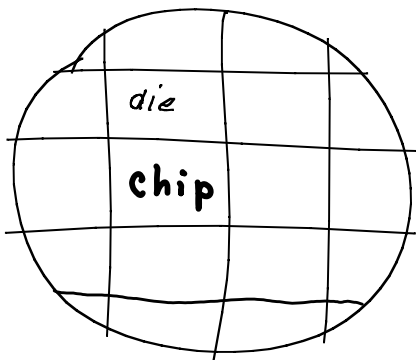




Line Sizes



Wafer

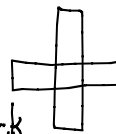


Better Process
more expensive
equipment
+ \$

lines shrink

10nm

faster switching
→ faster clock

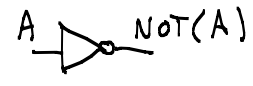
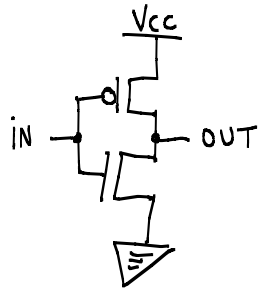


- more transistors
- more function
- more usage
- more sales
- same price

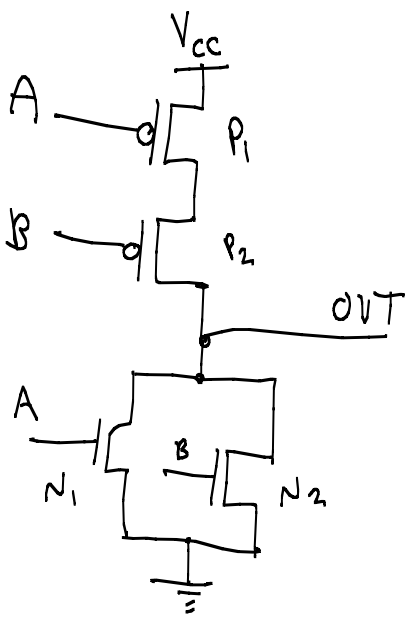
smaller features → more defects, lower die yield, but smaller die

Moore's Law

Basic Logic Gates



A	NOT(A)
0	1
1	0



A	B	P ₁	P ₂	N ₁	N ₂	OUT
0	1		 	 		0 _v
1	0	 			 	0 _v
0	0			 	 	V _{cc}
1	1	 	 			0 _v

A	B	out
0	0	1
0	1	0
1	0	0
1	1	0

NOT(OR)
NOR

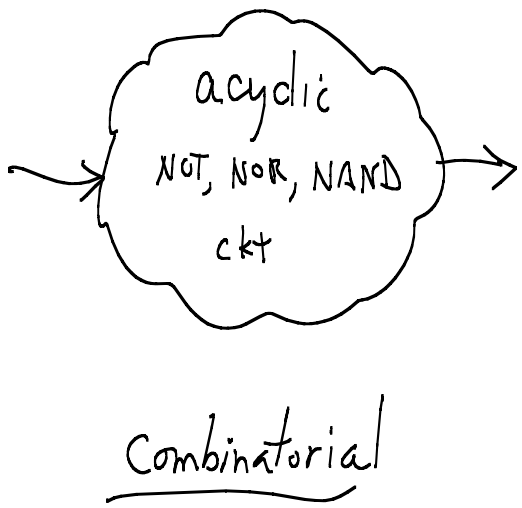
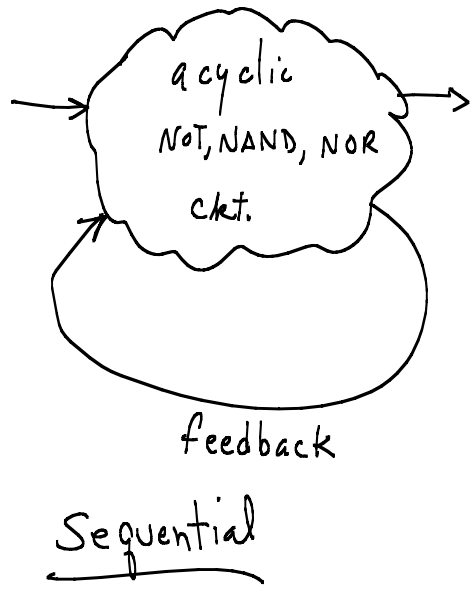
Logic Circuits

Two kinds of logic circuits:

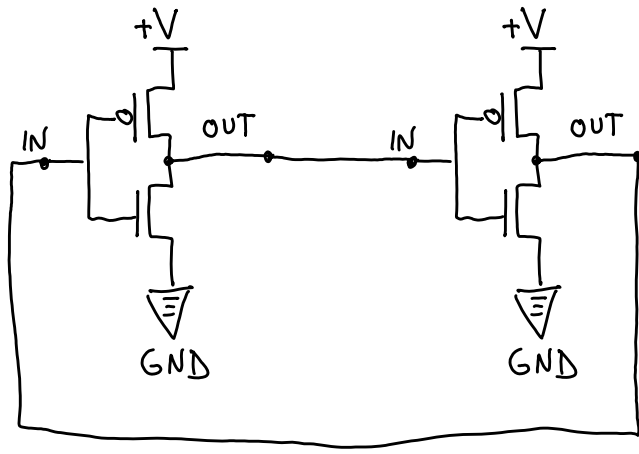
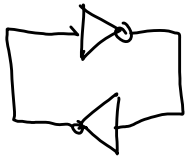
- (1) w/ feedback, SEQUENTIAL: can hold STATE
- (2) w/o feedback, COMBINATORIAL: realize FUNCTIONS

Basic logic gates: NOT, 2-input NOR, 2-input NAND.
That's all we need for both sequential and combinatorial circuits.

(NAND alone is sufficient, also NOR alone is sufficient.)

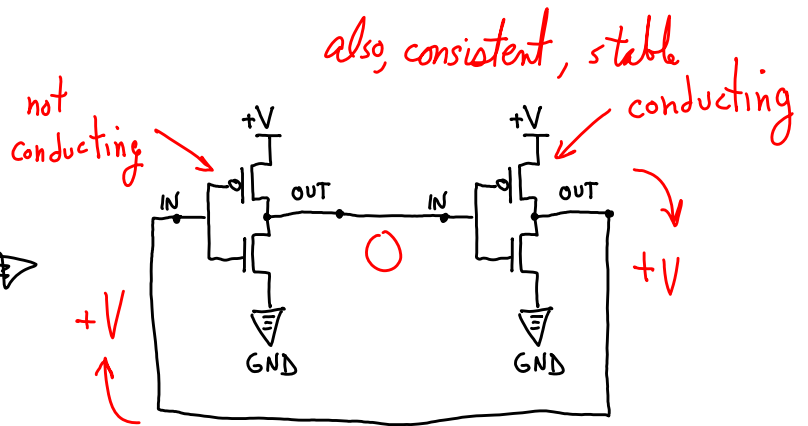
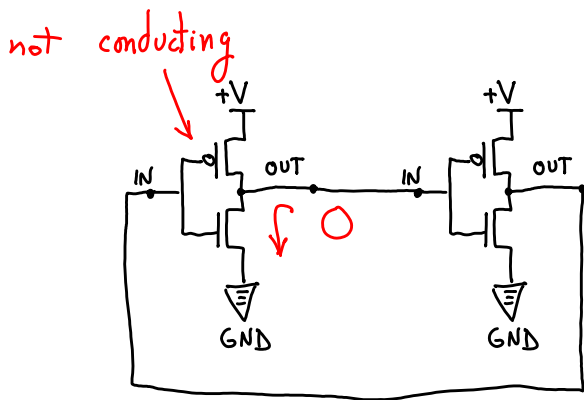
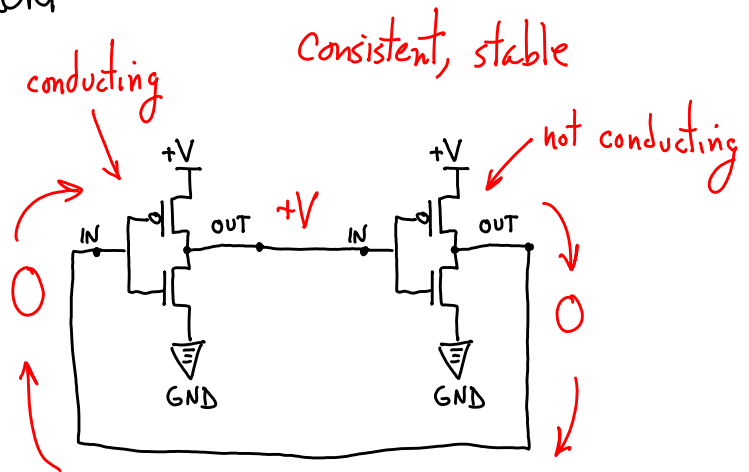
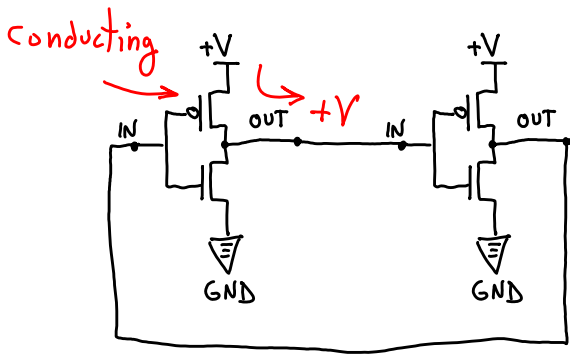


State feedback



what are the voltages?

Step 1 | Assume \rightarrow see if consistent

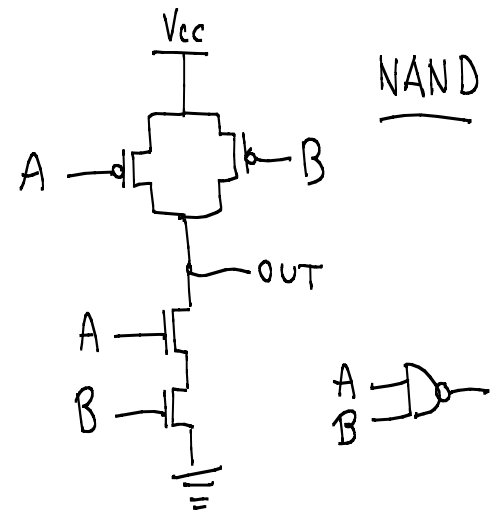
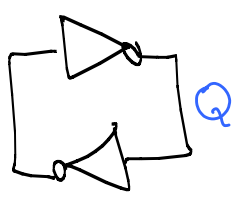


Circuit has two stable states: aka, meta-stable, bi-stable.

What state when power is first turned on? Unknown, random.

Can we set the state, using voltage inputs? No, useless!?!

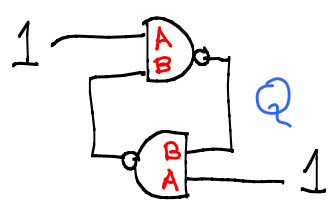
Basic Sequential elements



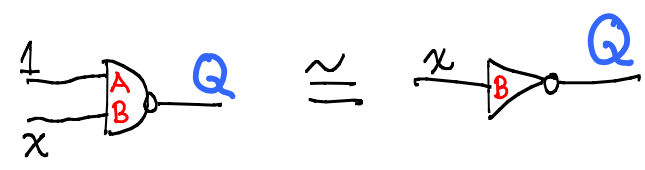
NAND

A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

A=1



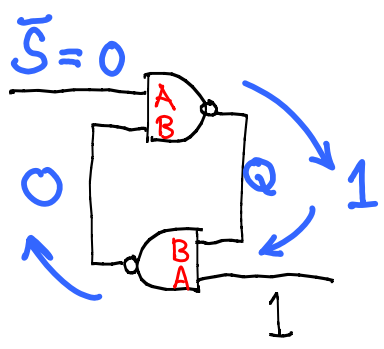
When $A=1$, $Q = \text{NOT}(B)$



CIRCUITS W/ STATE

NOT-NOT circuit is stable in either of two states: BISTABLE element.

NAND-NAND circuit with both A inputs = 1, same as a NOT-NOT circuit.

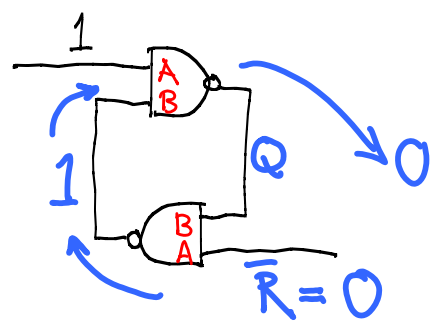


NAND

A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

A=0

$\Rightarrow Q=1$



$\bar{R} = \bar{S} = 1$: stable
 $\bar{R} = 1, \bar{S} = 0$: set $Q = 1$
 $\bar{R} = 0, \bar{S} = 1$: reset $Q = 0$

