Lec-6-HW-2-digitalDesign

Reading: PP-chp 3:

3.3 (decoder, mux, FA, PLA)
3.4 (R-S latch, register)
3.5 (memory)
3.6 (sequential machines, FSM)
3.7 (LC-3 datapath)

§ Problems, PP-chp 3:
§ 3.12 3-Dec, show minterm exp.
§ 3.13 5-dec, show num output lines.
§ 3.14 16X1 mux, how many select lines?
§ 3.19 explain mux ckt, s-r latch
§ 3.20 truth table => trans. Ckt -(-a.b)
§ 3.22 4X1 mux, from 2x1 MUXs
§ 3.24a 2x1 mux, identify input (select)
§ 3.25 gate delays for 32-Add
§ 3.27, simplified latch, figure out
§ 3.29 d-latch transparency
§ 3.30a 2-in-3-out comparator truth table.
§ 3.30b, logic ckt for (a).
§ 3.30c, 4-bit EQUAL from 1-bit 2X3 comparators
§ 3.31 #word X word_size = mem. Size
§ 3.32, addressability vs. address
§ 3.33a row X col addressing: find 4-th word
§ 3.33b #selects for 60 words?
§ 3.33c #words max for #select=3?
§ 3.43a fsm truth table
§ 3.43b state diagram for (a)

Question.

Assume P is a logical statement that is either true or false (P = 1 or P= 0).

Prove the following algebraic statements using truth tables. (The first is proved as an example.) You may also use the Duality Principle.

Extra Credit: Prove the Duality Principle. That is, prove that if two expressions are equivalent, then their dual expressions are equivalent.

$1 \cdot P = P$

Proof: here is the truth table for AND,

1	P	$1 \cdot P$
1	0	0
1	1	1

where 1 is the constant function

Here's the truth table for P,

1	P	P
1	0	0
1	1	1

They have the same output column, therefore, they are equivalent.

$0 + P = P$

$0 \cdot P = 0$

$1 + P = 1$

$P \cdot P = P$

$P + P = P$

$P \cdot \overline{P} = 0$

$P + \overline{P} = 1$

Question.

Assume A and B and C are logical statements that are either true or false (1 or 0).

Prove the following algebraic statements using truth tables.

$(\cdot +)$-distributivity: $A \cdot (B + C) = A \cdot B + A \cdot C$

$(+ \cdot)$-distributivity: $A + (B \cdot C) = (A + B) \cdot (A + C)$

DeMorgan's Law I: $\overline{A \cdot B} = \overline{A} + \overline{B}$

DeMorgan's Law II: $\overline{A + B} = \overline{A} \cdot \overline{B}$

Question.

Try to find the simplest (that is, fewest gates needed) form of an expression for *f*. Justify your derivation of the expression using the algebraic properties we have proven above. The first is solved as an example.

A	B	f	
0	0	0	
0	1	1	$\rightarrow \bar{A}B$
1	0	0	
1	1	1	$\rightarrow AB$

$$f = \bar{A}B + AB$$

$$= (\bar{A} + A)B \quad \text{from distributivity}$$

$$= 1 \cdot B \quad \text{from } p + \bar{p} = 1$$

$$= B \quad \text{from } 1 \cdot p = p$$

A	B	f
0	0	1
0	1	1
1	0	0
1	1	1

A	B	C	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Question .

Use DeMorgan's Laws to find a maxterm expression for the function *f*.
(The first is solved as an example.)

find a minterm expansion for $\bar{f}$. Then, convert it to an expression for f using DeMorgan's Laws.

A	B	f(A, B)
0	0	0
0	1	1
1	0	1
1	1	0

A	B	$\overline{f(A,B)}$	
0	0	1	$\longrightarrow \bar{A}\cdot\bar{B}$
0	1	0	
1	0	0	
1	1	1	$\longrightarrow A\cdot B$

$$\bar{f} = \bar{A}\cdot\bar{B} + A\cdot B$$

$$\bar{\bar{f}} = \overline{(\bar{A}\bar{B}) + (AB)} \quad \text{II}$$

$$= \overline{(\bar{A}\cdot\bar{B})}\cdot\overline{(AB)} \quad \text{I}$$

$$= (\bar{\bar{A}} + \bar{\bar{B}})\cdot(\bar{A} + \bar{B})$$

since $\bar{\bar{f}} = f$, this last expression is a maxterm expression for $f = (A+B)\cdot(\bar{A}+\bar{B})$

A	B	C	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

A	B	f
0	0	0
0	1	1
1	0	1
1	1	0