1 Discrete Timesteps

Single drive

P(alive at time slot t) = p

P(fail at time slot t+1) = $p^t q$

$$\begin{aligned}
\text{MTTF} = &\sum_{t=0}^{\infty} tp^t q = q\sum_{t=0}^{\infty} p\frac{dp^t}{dp}\\
=&qp\frac{d}{dp}\sum_{t=0}^{\infty} p^t = qp\frac{d}{dp}\frac{p}{1-p}\\
=&qp(\frac{1}{1-p}+\frac{p}{(1-p)^2})\\
=&p/q
\end{aligned}$$

As proven in class.

I couldn't figure out how to solve MTTF for 1 element out of 2, with normalisation factor K, where $P(\text{1 fails at time slot t and other is alive}) = K * p^t q * \sum_{t_1=t+1}^{\infty} p^{t_1} q$

2 Standard integrals used below

$$\int_{x=0}^{\infty} e^{-cx}dx = \frac{1}{c} \tag{1}$$

$$\int_{x=0}^{\infty} xe^{-cx}dx = \frac{1}{c^2} \tag{2}$$

$$\int xe^{-cx}dx = \frac{e^{cx}}{c^2}(cx-1) \tag{3}$$

3 Continuous Time

Exponential Decay equation – $N(t) = N_0 e^{-\lambda t}$

Converting to a probability distribution for single object

$P(\text{fail at time } t) = \lambda e^{-\lambda t}$ - from equation 1

Let $TTF_{N,M}(t)$ = Probability that the Nth objects fails, out of M total objects, at time t.

$TTF_{1,1}(t) = P(\text{fail at time } t)$

$MTTF_{1,1} - \tau = \frac{1}{\lambda}$ – use equation 2 on $TTF_{1,1}(t)$

Let K be the normalisation factor

$$\begin{aligned}
TTF_{1,2}(t) =& K * P(\text{obj 1 fails at time } t) * P(\text{obj 2 fails at time } t_1 > t) \\
=& K\lambda e^{-\lambda t} \int_t^\infty \lambda e^{-\lambda t_1} dt_1 \\
=& K\lambda e^{-\lambda t} * e^{-\lambda t} \\
=& K\lambda^2 e^{-2\lambda t}
\end{aligned}$$

Integrating over $0 < t < \infty$ for normalization yields $TTF_{1,2}(t) = 2\lambda e^{-2\lambda t}$

Therefore,

$$\begin{aligned}
MTTF_{1,2} =& \int_0^\infty 2\lambda t e^{-2\lambda t} dt \\
=& 2\lambda \frac{1}{(2\lambda)^2} \\
=& \frac{1}{2\lambda}
\end{aligned}$$

By corollary,

$$
\begin{aligned}
MTTF_{1,N} = &\int_0^\infty N\lambda t(e^{-\lambda t})^N dt \\
=&N\lambda \int_0^\infty te^{-\lambda N t} dt \\
=&N\lambda \frac{1}{N^2\lambda^2} \\
=&\frac{1}{N\lambda}
\end{aligned}
$$

Given some prob distribution for the Time To Recover (TTR), the time to total failure (TTTF) is the probability that the time to recover is larger than the time between obj1 failing and obj2 failing. As an equation, this can be expressed, with normalisation factor K, as
$TTTF_2(t) = K * P(\text{obj1 fails at time } t) * P(\text{obj2 fails at time } t_1 < t) * P(\text{time to recover} > t - t_1)$

As an example, let $TTR = \mu e^{-\mu t}$

$$
\begin{aligned}
TTTF_2(t) =&K\lambda e^{-\lambda t} \int_0^t \lambda e^{-\lambda t_1} \int_{t-t_1}^\infty \mu e^{-\mu t_2}\ dt_2\ dt_1 \\
=&\frac{K\lambda^2}{\mu - \lambda}(e^{-2\lambda t} - e^{-(\lambda+\mu)t})
\end{aligned}
$$

Integrating to find the normalisation constant gives $K = \frac{2(\lambda+\mu)}{\lambda}$

$$
TTTF_2(t) = \frac{2\lambda(\lambda+\mu)}{\mu - \lambda}(e^{-2\lambda t} - e^{-(\lambda+\mu)t})
$$

Integrating using the equation 2 gives MTTTF (Mean Time To Total Failure)

$$
\begin{aligned}
MTTTF_2(t) &= \int_0^\infty \frac{2\lambda(\lambda+\mu)}{\mu-\lambda} t(e^{-2\lambda t} - e^{-(\lambda+\mu)t})\ dt \\
&= \frac{2\lambda(\lambda+\mu)}{\mu-\lambda} \int_0^\infty t(e^{-2\lambda t} - e^{-(\lambda+\mu)t})\ dt \\
&= \frac{2\lambda(\lambda+\mu)}{\mu-\lambda} \left(\frac{1}{4\lambda^2} - \frac{1}{(\lambda+\mu)^2} \right) \\
&= \frac{1}{2\lambda} + \frac{1}{\lambda+\mu}
\end{aligned}
$$

Other TTR distributions will give different results, but may not yield a nice final expression.