- Bandwidth or throughput
  - Total work done in a given time
  - 10,000-25,000X improvement for processors
  - 300-1200X improvement for memory and disks

- Latency or response time
  - Time between start and completion of an event
  - 30-80X improvement for processors
  - 6-8X improvement for memory and disks

Performance

Comparing Machines/Systems

- Response Time (latency) = T
  - How long does it take for my job to run?
  - How long does it take to execute a job?
  - How long must I wait for the database query?

avg/best case/worst case

- Throughput = ∀
  - How many jobs can the machine run at once?
  - What is the average execution rate?
  - How many queries per minute?

avg.

What do we "really" want to know?

--- Which system works best in our larger system?

--- What costs can be traded off?

Time?

- Elapsed Time
  - Counts everything *(disk and memory accesses, I/O , etc.)* → "wall clock"
  - A useful number, but often not good for comparison purposes
    - E.g., OS & multiprogramming time make it difficult to compare CPUs

Depends on load, disk layout, ...

- CPU time (CPU = Central Processing Unit = processor)
  - Doesn't count I/O or time spent running other programs
  - Can be broken up into system time, and user time → user cpu time

OS
I/O

- Our focus: user CPU time
  - Time spent executing the lines of code that are "in" our program
  - Includes arithmetic, memory, and control instructions...

→ Time CPU used for our job (+ overhead)

unix ⇒ localhost > time myJob

more abstract

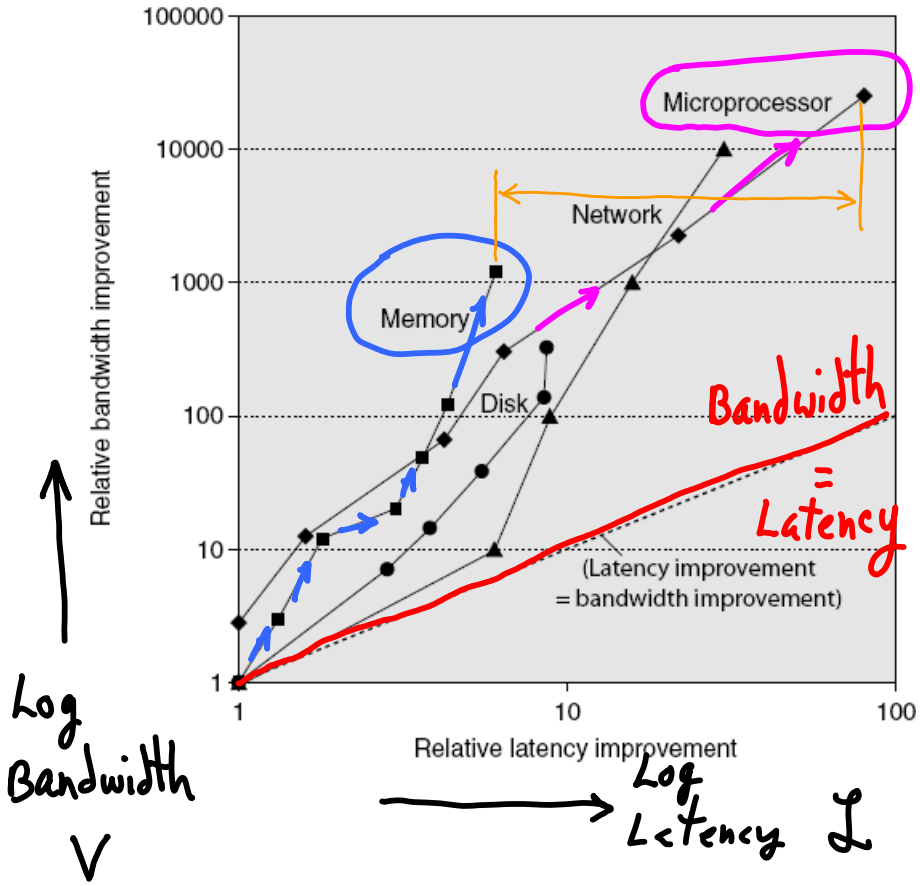
Latency vs. Bandwith

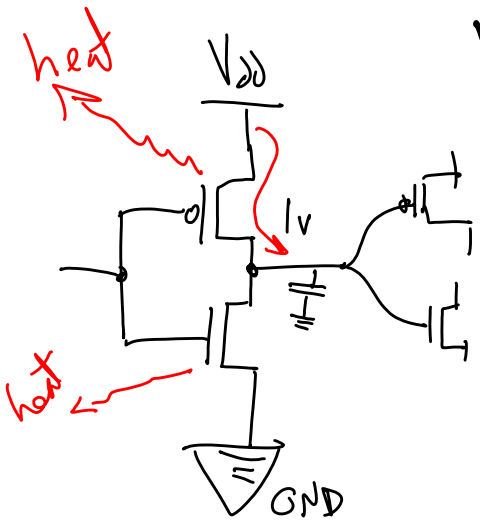
Increasing complexity/density/speed

1. V / L gets worse for each component.

2. Vproc / Lmem gets worse even faster

relative performance





- Dynamic energy
  - Transistor switch from 0 -> 1 or 1 -> 0
  - ½ x Capacitive load x Voltage^2
- Dynamic power
  - ½ x Capacitive load x Voltage^2 x Frequency switched
- Reducing clock rate reduces power, not energy

$$f_{max} = \propto V$$

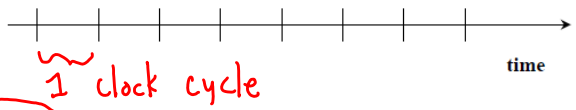
cpu Clock Cycles ⇒ cpu time

- Instead of reporting execution time in seconds, we often use cycles

CPU Time = $\left(\frac{\text{seconds}}{\text{program}}\right) = \left(\frac{\text{cycles}}{\text{program}}\right) \times \left(\frac{\text{seconds}}{\text{cycle}}\right)$

$T_{cycle} = \left(\frac{\text{seconds}}{\text{Tick}}\right)$

- Clock "ticks" indicate when to start activities:



1 clock cycle

$Freq = \left(\frac{\text{Ticks}}{\text{sec}}\right) = \frac{1}{T_{cycle}}$

- Cycle time = time between ticks = seconds per cycle
- Clock rate (frequency) = cycles per second (1 Hz. = 1 cycle/sec)

2 GHz clock ⇒ $Freq = \left(\frac{2 \times 10^9 \text{ ticks}}{\text{sec}}\right)$ ⇒ $T_{cycle} = \left(\frac{1 \text{ sec}}{2 \times 10^9 \text{ Ticks}}\right)$

$= \frac{1}{2}$ ns

$= \frac{1}{2}(1{,}000 \text{ ps})$

$= 500$ ps

10^{-12} sec

CR ≡ Freq

- User *CPU execution time*

$$\text{Execution Time} = \text{Clock Cycles for Program} \times \text{Clock Cycle Time}$$

※ cycles × T_{cycle}

rewrite

- Since Cycle Time is 1/Clock Rate (or clock frequency)

$$\text{Execution Time} = \frac{\text{Clock Cycles for Program}}{\text{Clock Rate}} = ※\text{cycles} \left(\frac{1}{T_{cycle}}\right)^{-1}$$

$= \frac{※\text{cycles}}{CR}$

- The program should be something real people care about
  - Desktop: MS office, edit, compile
  - Server: web, e-commerce, database
  - Scientific: physics, weather forecasting

} real jobs? or benchmarks?

Measuring Clock Cycles

- Clock cycles/program is not an intuitive or easily determined value, so

$$\#\ Clock\ Cycles = Instructions \times Clock\ Cycles\ Per\ Instruction$$

$$= \#\,instr \times \left(avg\ \frac{\#\,cycles}{instr}\right)$$

$$= \sum_{i=1}^{\#\,instr} \#\,cycles_i$$

each instruction is different

- Cycles Per Instruction (CPI) used often
- CPI is an **average** since the number of cycles per instruction varies from instruction to instruction
  - Average depends on instruction mix, latency of each inst. type etc.
- CPIs can be used to compare two implementations of the same ISA, but is not useful alone for comparing different ISAs
  - An x86 add is different from a MIPS add

$CPI_{x86} \neq CPI_{mips}$

[lc3: ADD vs RTI]

Depends on instruction mix, system configuration, data

$(\#\,instr \times CPI) \quad 1/CR$

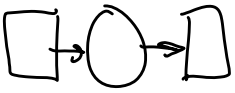
$= \quad =$

$\#\,cycles \times T_{cycle}$

- Drawing on the previous equation:

$$Execution\ Time = Instructions \times CPI \times Clock\ Cycle\ Time$$

$$Execution\ Time = \frac{Instructions \times CPI}{Clock\ Rate} = \#\,instr\left(\frac{CPI}{CR}\right)$$



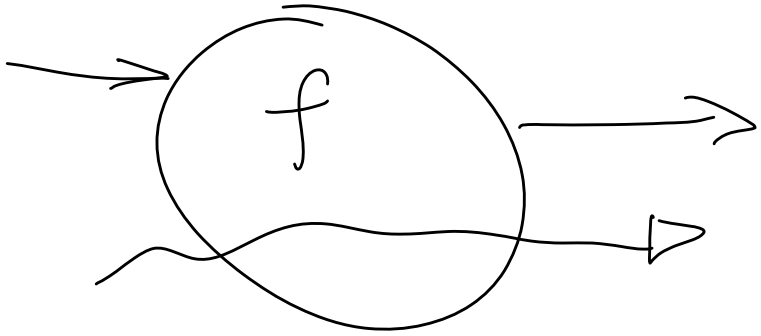
- To improve performance (i.e., reduce execution time)
  - Increase clock rate (decrease clock cycle time) OR
  - Decrease CPI OR
  - Reduce the number of instructions
- Designers balance cycle time against the number of cycles required
  - Improving one factor may make the other one worse...

Time ↓

CR ↑ ⇒ CPI ↓ ⇒ #instr ↑

shallow logic — simpler instr — more instructions

[look for sweet spot]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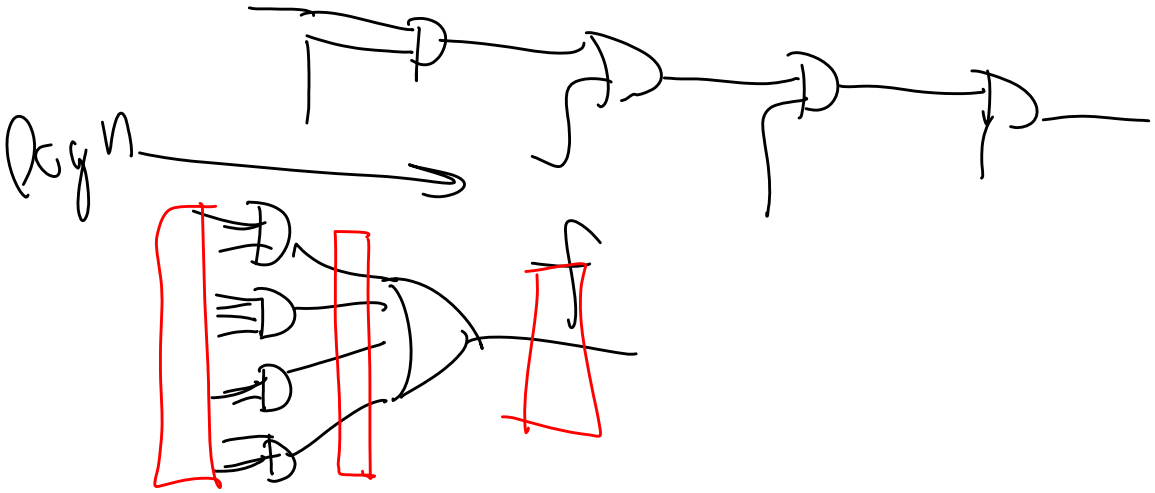
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Clock Rate ≠ Performance

- Mobile Intel Pentium 4 Vs Intel Pentium M
  - 2.4 GHz 1.6 GHz
  - P4 is 50% faster?

$$\frac{CR_{P4}}{CR_{PM}} = \frac{2.4}{1.6} = 1.5$$

- Performance on Mobilemark with same memory and disk
  - Word, excel, photoshop, powerpoint, etc.
  - But Mobile Pentium 4 is only 15% faster

Instr. Count

time = (# instr) $\left(\frac{CPI}{CR}\right)$

- What is the relative CPI?
  - ExecTime = IC • CPI/Clock rate
  - $\text{ExecTime}_M = 1.15\ \text{ExecTime}_4$
  - $IC \bullet CPI_M/1.6 = 1.15 \bullet IC \bullet CPI_4/2.4$
  - $CPI_4/CPI_M = 2.4/(1.15 \bullet 1.6) = 1.3$

$$T_{P4} = IC_{P4}\left(\frac{CPI_{P4}}{CR_{P4}}\right)$$

$$T_{PM} = IC_{PM}\left(\frac{CPI_{PM}}{CR_{PM}}\right)$$

$$T_{P4}(1.15) = T_{PM}$$

$$\left(IC_{P4}\left(\frac{CPI_{P4}}{CR_{P4}}\right)\right)(1.15) = IC_{PM}\left(\frac{CPI_{PM}}{CR_{PM}}\right)$$

Same ISA

$$IC_{P4} = IC_{PM}$$

and

$$CR_{P4} = (1.5)\,CR_{PM}$$

$$\frac{CPI_{P4}}{(1.5)\,CR_{PM}}(1.15) = \frac{CPI_{PM}}{CR_{PM}}$$

rearrange

$$\frac{CPI_{P4}}{CPI_{PM}} = \frac{(1.5)}{(1.15)} = 1.304\ldots \Rightarrow$$ 30% more cycles/instr on avg for P4

Break it down by classes

- Different instruction types require different numbers of cycles
- CPI is often reported for types of instructions

$$Clock\ Cycles = \sum_{i=1}^{n} (CPI_i \times IC_i)$$

n = # types

IC_i = # (type i instructions)

CPI_i = (avg CPI for type i instructions)

- where CPI_i is the CPI for the type of instructions and IC_i is the count of that type of instruction

- To compute the overall average CPI use

$$\frac{\#\ cycles}{IC} \quad CPI = \sum_{i=1}^{n} \left(CPI_i \times \frac{Instruction\ Count_i}{Instruction\ Count} \right)$$

$$IC = \sum_{i=1}^{n} IC_i$$

$\#\ Cycles_i$

$$\frac{1}{IC} \left(\sum_{i=1}^{n} (CPI_i \times IC_i) \right)$$

$$= \sum_{i=1}^{n} CPI_i \times \frac{IC_i}{IC}$$

$$= \sum CPI_i \times \%_i$$

$= avg\ CPI$

CPI_i $\quad$ $\%_i$ $\Rightarrow \{ CPI_i (\%_i) \}$ i^{th} contribution to overall avg CPI

Instruction Type	CPI	Frequency	CPI * Frequency
ALU	1	50%	0.5
Branch	2	20%	0.4
Load	2	20%	0.4
Store	2	10%	0.2

SUM = 1.5 = (avg) CPI

- Given this machine, the CPI is the sum of CPI X Frequency
- Average CPI is 0.5 + 0.4 + 0.4 + 0.2 = 1.5

$$= \sum_i \#(cycles\ for\ type\ i) / IC$$

$$= \#\ cycles / IC$$

$$= avg\ CPI$$

- What fraction of the time for data transfer?

$$\frac{T_{LD-ST}}{T_{Total}} = \frac{Cycles_{LD-ST} * (1/f)}{Cycles_{Total} * (1/f)} = \frac{\#(cycles\ LD\text{-}ST)}{\#\ cycles} = \frac{(\#cycles\ LD) + (\#cycles\ ST)}{(\#\ cycles) = IC \times CPI}$$

cycle Time

$$= \left[\frac{(\#cycles\ LD)}{IC} + \frac{(\#\ cycles\ ST)}{IC}\right] \frac{1}{CPI}$$

$$= \left[\frac{CPI_{LD} \cdot IC_{LD}}{IC} + \frac{CPI_{ST} \cdot IC_{ST}}{IC}\right] \frac{1}{CPI}$$

$$= [CPI_{LD} \cdot \%_{LD} + CPI_s \cdot \%_{ST}]\left(1/1.5\right)$$

$$= [0.4 + 0.2](1/1.5) = 40\%$$

Speedup

- Speedup allows us to compare different CPUs or optimizations

$$Speedup = \frac{CPUtimeOld}{CPUtimeNew}$$

- Example
  - Original CPU takes 2sec to run a program
  - New CPU takes 1.5sec to run a program
  - Speedup = 1.333 or speedup or 33%

$T_{old} = 2\,s$

$T_{new} = 1.5\,s$

$$S_{new-old} = T_{old}/T_{new} = 2/1.5 = \frac{20}{15} = 1 + 5/15 = 1.3\ldots$$

$$\Rightarrow \frac{T_{old}}{1.3} = T_{new} \ (?) \quad S = 1/1.3 \ (?)$$

Does this look like speedup? What do we mean by speedup?

$$S = \frac{V_{new}}{V_{old}} = \frac{(W_{new}/T_{new})}{(W_{old}/T_{old})} = T_{old}/T_{new} \Rightarrow V_{new} = V_{old}\,(1.3)$$

$\Rightarrow$ new is 30% faster

Assuming $W_{old} = W_{new}$

$S = \frac{Q_{old}}{Q_{new}} = \frac{W/T_{old}}{W/T_{new}}$

$W = W_{parallel} + W_{sequential} = f \cdot W + (1-f)W$

$T = T_{parallel} + T_{sequential}$ (parall = improvable, sequential = fixed)

$Q_p = W_p/T_p \quad Q_s = W_s/T_s$

old: $Q_p = Q_s$

new: $Q_p = a Q_s$ (ie. $S_p = a$)

Amdahl's Law

- If an optimization improves a fraction f of execution time by a factor of a

$$Speedup = \frac{Told}{[(1-f)+f/a]*Told} = \frac{1}{(1-f)+f/a}$$

Is f fixed? $W_p = \Theta(n)$ $W_s = \Theta(1)$

- This formula is known as Amdahl's Law
- Lessons from
  - If f→100%, then speedup = a
  - If a→∞, the speedup = 1/(1-f)
- Summary
  - Make the common case fast (target speed up to max %)
  - Watch out for the non-optimized component

$T_{old} = \frac{f \cdot W}{Q_s} + \frac{(1-f)W}{Q_s} = W/Q_s$

$T_{new} = W_p/Q_p + W_s/Q_s = \frac{f \cdot W}{a Q_s} + \frac{(1-f)\cdot W}{Q_s} = \left(\frac{f}{a} + (1-f)\right) W/Q_s$

$S_{new-old} = \frac{T_{old}}{T_{new}} = \frac{(W/Q_s)}{(W/Q_s)(f/a + (1-f))} = 1/(f/a + (1-f))$

$f=1 \rightarrow 1/(1/a + 0) = a = S_{all\text{-}improved} = S_{max}$

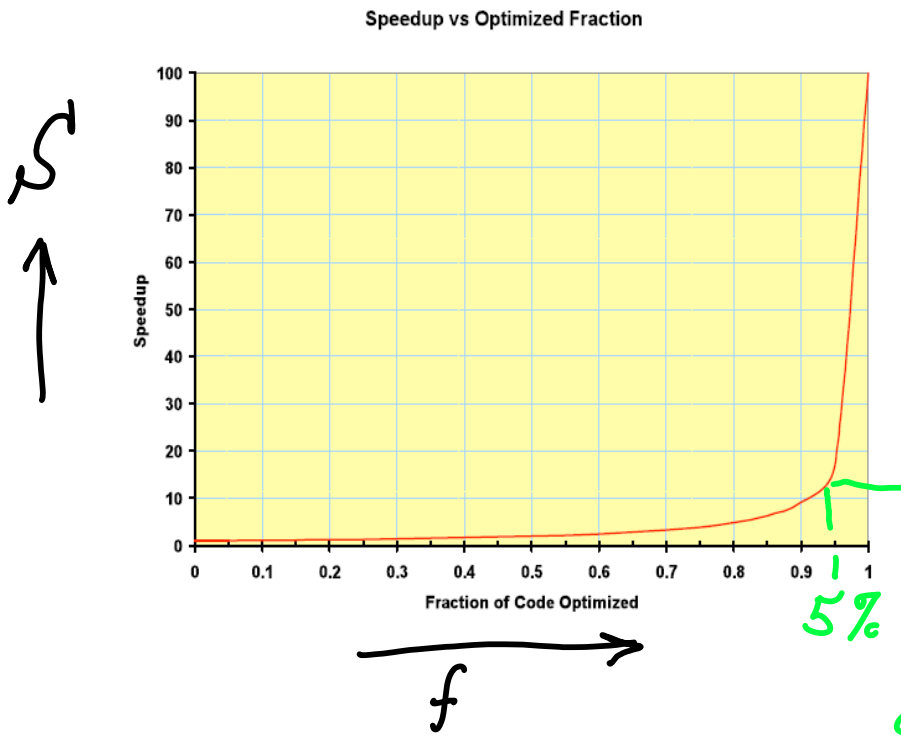
$f=0 \rightarrow 1/(0/a + 1) = 1 = S_{none\text{-}improved} = S_{min}$

$a \rightarrow \infty \rightarrow 1/(f/\infty + (1-f)) = \frac{1}{(1-f)} = S_\infty \geq S_{actual}$

$f = 50\% \Rightarrow S_\infty = \frac{1}{(1-1/2)} = 2$ max improvement for $a \rightarrow \infty$ is twice as fast

- If a=100, what is the overall speedup as a function of f?

e.g., Use 100 CPUs for $W_{parallel}$



$S = 17$?

5% sequential

$cost_{new} = 100 \cdot Cost_{old}$

Amdahl's Law Example

- Suppose a program runs in 100 seconds on a machine, with multiply responsible for 80 seconds of this time. How much do we have to improve the speed of multiplication if we want the program to run 4 times faster?" $= S = 4$

$T_{old} = T_{other} + T_{mult} = 20_s + 80_s = 100_s$

find $S_p = \frac{T_{p\,old}}{T_{p\,new}} = ?$

- How about making it 5 times faster? $S = 5$?

$S_p = \frac{T_p^{old}}{T_p^{new}} = \frac{80}{5} = 16$

$x \cdot 7 = x(4+2+1) = 4x + 2x + x$

$y = 00\ldots0111$

4x: 2 shifts (C); 2x: 1 shift (B); x: 0 shifts (A)

$$\begin{array}{rl} & 0\ 0\ \cdots\ 0\ 0 \\ + & x_n\ x_{n-1}\ \cdots\ x_1\ x_0 \\ \hline & s'_n\ s'_{n-1}\ \ldots\ s'_1\ s'_0 \end{array} \quad = S^0 \quad + (A) \quad S'$$

shift

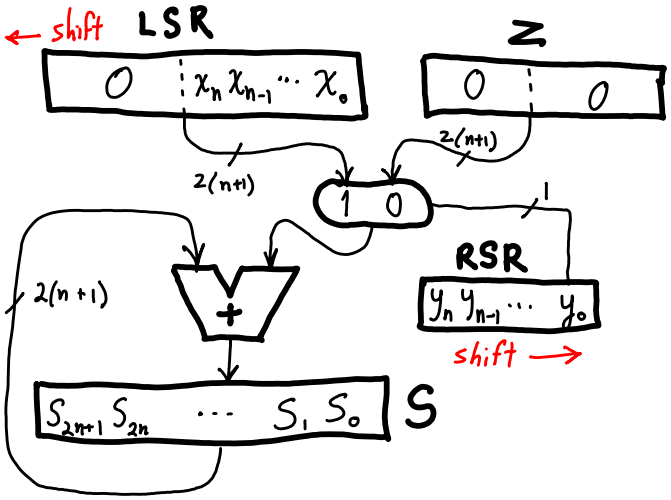
$$\begin{array}{rl} + & x_n\ x_{n-1}\ \cdots\ x_1\ x_0\ 0 \\ \hline & s''_{n+1}\ s''_n\ \cdots\ s''_2\ s''_1\ s''_0 \end{array} \quad + (B) \quad S''$$

shift

$$\begin{array}{rl} + & x_n\ x_{n-1}\ \cdots\ x_1\ x_0\ 0\ 0 \\ \hline & s'''_{n+2}\ s'''_{n+1}\ \cdots\ s'''_3\ s'''_2\ s'''_1\ s'''_0 \end{array} \quad + (C) \quad S'''$$

What if y has a 0 bit? Then add 0 instead of shifted x: e.g., y = 0...101 add 0, not B.

INT MULTIPLY: S = x * y
LSR: partial products, initially x.
S: partial sum, initially 0.
RSR: initially y.
Z: all 0s



e.g.

$$\begin{array}{rl} & 1011 \\ \times & 0101 \\ \hline & \ldots1011 \\ + & .1011.. \\ \hline & 00111011 \end{array}$$

(possible carry)

rewrite

$$\begin{array}{rl} & 1011 \\ \times & 0101 \\ \hline & 00000000 \quad \leftarrow \text{start} \\ + & 00001011 \quad \leftarrow \text{0-shift (1011)} \quad \leftarrow Y_0 = 1 \\ + & 00000000 \quad \leftarrow \text{1-shift ( )} \quad \leftarrow Y_1 = 0 \\ + & 00101100 \quad \leftarrow \text{2-shift ( )} \quad \leftarrow Y_2 = 1 \\ + & 00000000 \quad \leftarrow \text{3-shift ( )} \quad \leftarrow Y_3 = 0 \\ \hline = & 00111011 \end{array}$$

multiplier bit

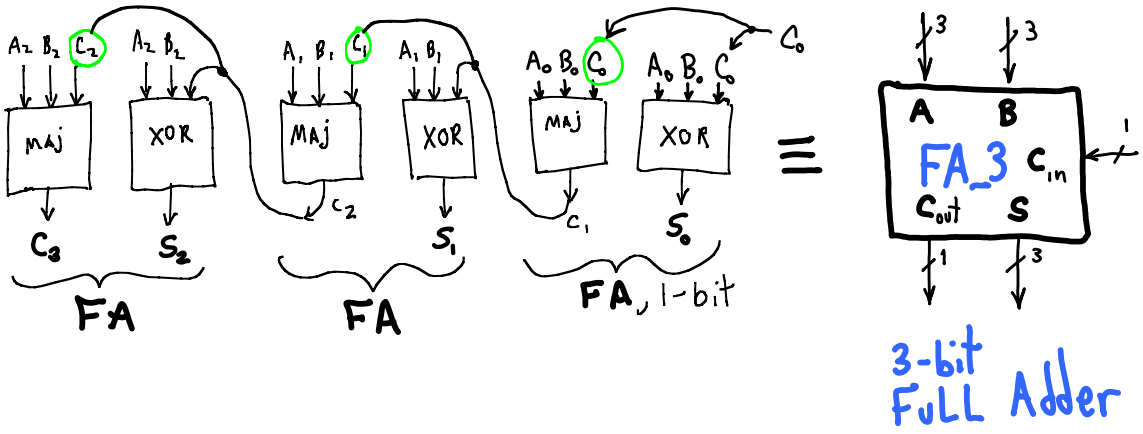
⇒ we shift left (1011) every time, but add either the shifted (1011) or all zeroes, depending on whether Y_i is 1 or 0.

ADDER

Delay is longest path.

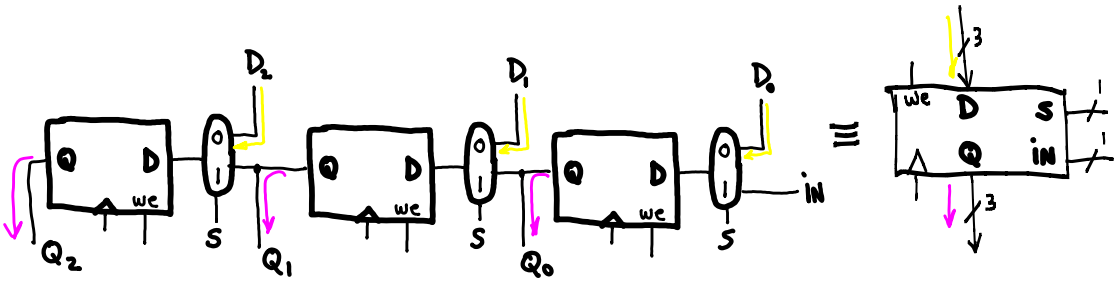
2 levels per MAJ, n bit operands.

2n gate delays until result is ready.



3-bit FULL Adder

Shift Register Delay = 1 (all stages in parallel)

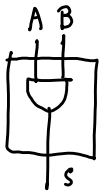


Overall delay: (n shifts) × (2n delay$_{ADDER}$) = $2n^2$

Improvement?	Recursive Refinement?

(1-bit MULT)

$$\begin{array}{cccc} 0 & 0 & 1 & 1 \\ \times 0 & \times 1 & \times 0 & \times 1 \\ \hline 0 & 0 & 0 & 1 \end{array} \Rightarrow$$

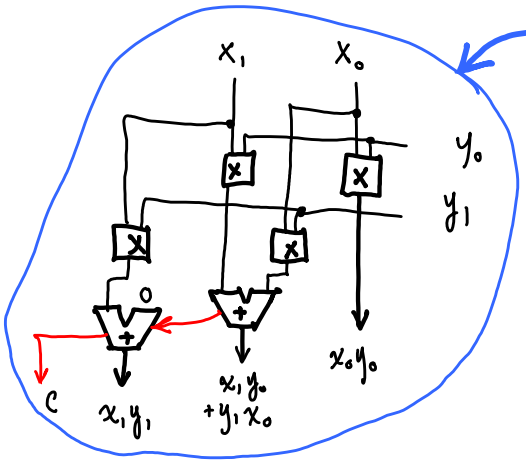


$=$ [X]

(2-bit MULT)

$$\begin{array}{r} (x_1\ x_0) \\ \times\ (y_1\ y_0) \\ \hline c\ \ c \\ 0\ \ 0\ \ x_0y_0 \\ +\ 0\ \ x_1y_0\ \ 0 \\ +\ 0\ \ y_1x_0\ \ 0 \\ +\ x_1y_1\ \ 0\ \ 0 \\ \hline S_3\ \ S_2\ \ S_1\ \ S_0 \end{array}$$

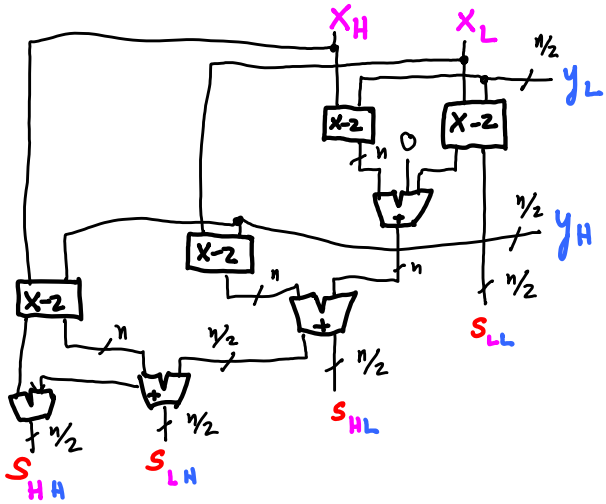
$=$



(Use Recursively)

(n-bit)

$$\begin{array}{rcccc} & [x_3x_2] & & [x_1x_0] & \\ \times & [y_3y_2] & & [y_1y_0] & \\ \hline c\ \ldots & c & & & \\ 0 & 0 & [x_L \cdot y_L] & & \\ 0 & [x_H \cdot y_L] & & 0 & \\ 0 & [x_L \cdot y_H] & & 0 & \\ + & [x_H \cdot y_H] & 0 & 0 & \\ \hline & S_{HH} & S_{LH} & S_{HL} & S_{LL} \end{array}$$



$\Rightarrow$ longest delay, $f(n)$, is,

4n-bit Add + (n/2)-bit MULT delay:

$$f(n) = 4(2n) + f(n/2)$$

$$f(n) = 8n + f(n/2) = 8n + 8(n/2) + f(n/4) \Rightarrow 16n\left(\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \cdots\right) \approx 16n$$

time improvement: $2n^2 \Rightarrow 16n$ Area increase: $7n \Rightarrow ?$ $A(n) = 4A(n/2) + 4(n)$

32-bit $2(2^5)^2 = 2^{11} \Rightarrow 2^4(2^5) = 2^9$ $S_{32\text{-bit}} = 2^{11}/2^9 = 2^2 = 4$

$S_{64\text{-bit}} = 8$

$S_{128\text{-bit}} = 16$

$$4 = S_{overall} = \frac{q_{new}}{q_{old}} = \frac{W/T_{new}}{W/T_{old}} = \frac{T_{old}}{T_{new}} = \frac{W_s/V_{s\text{-}old} + W_p/V_{p\text{-}old}}{W_s/V_{s\text{-}new} + W_p/V_{p\text{-}new}}$$

OUR Requirement

$$= \frac{(0.2)W/V_{p\text{-}old} + (0.8)W/V_{p\text{-}old}}{(0.2)W/V_{p\text{-}old} + (0.8)W/S_p \cdot V_{p\text{-}old}}$$

[assume $V_{p\text{-}old} = V_{s\text{-}old} = V_{s\text{-}new}$, $V_{p\text{-}new} = S_p V_{p\text{-}old}$]

$$= 1/(0.2 + 0.8/S_p) = 4$$

$$\Rightarrow 0.25 = 0.2 + 0.8/S_p$$

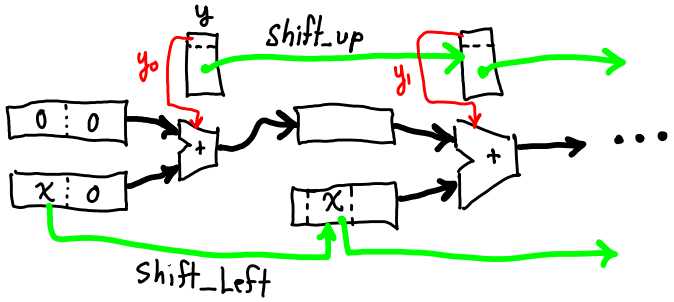
$$\Rightarrow 0.05 = 0.8/S_p \quad \Rightarrow S_p = \frac{0.8}{0.05} = \frac{80}{5} = 16 = S_{128\text{-MULT}}$$

OK! We can just make it if our data is 128-bit

$S_{overall} = 5$? $1/(0.2 + 0.8/S_p) \stackrel{?}{=} 5$ Can we?

$S_{overall-\infty} = 1/(0.2 + 0.8/\infty) = 1/0.2 = 5$ Possible? if we eliminate MULT time?

what else could we try?



n-stage pipeline

1 result every shift

delay = $4n$ (2n-bit ADD)

(32-bit):

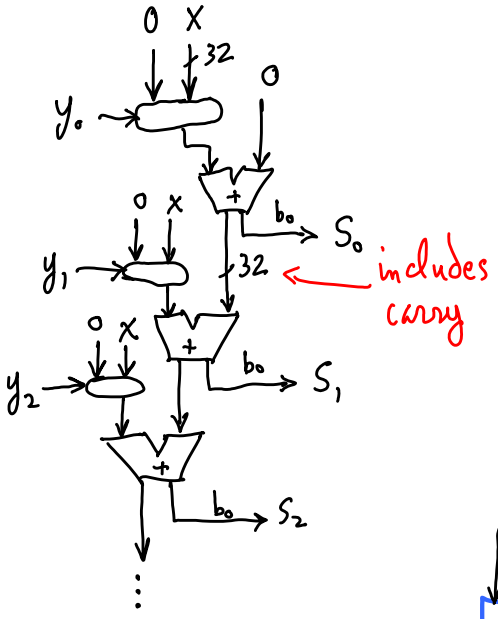
$$S_{32\text{-MULT}} = \frac{2(2^5)^2}{4(2^5)} = 2^4 = 16$$

Can we keep it full?

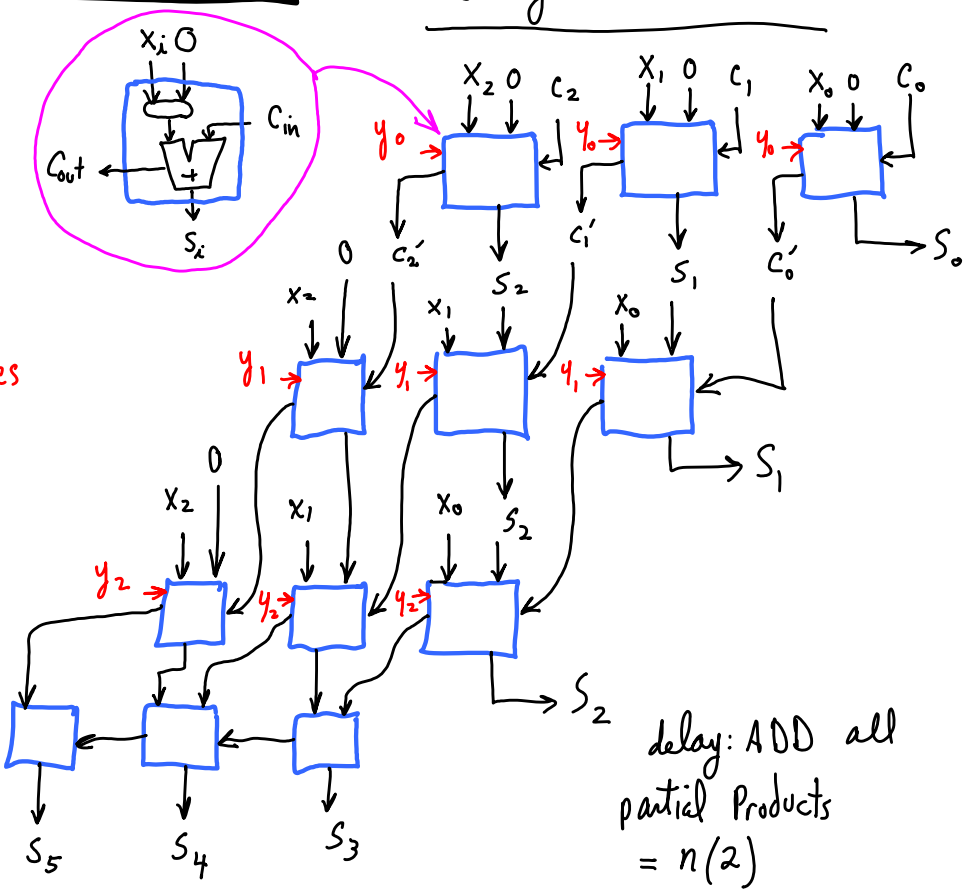
Latency = $4n \times (n) = 4n^2$

Another Approach? Back to square 1

Adding Partial Products



carry-save adder



delay: ADD all partial Products = $n(2)$

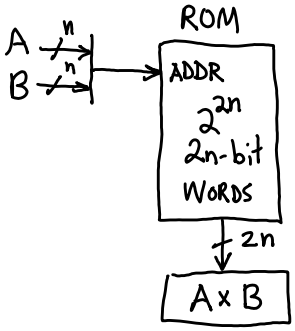
$$S_{32\text{-MULT}} = 2^{11}/_{26} = 32$$

$$S = 1/(0.2 + 0.8/32)$$

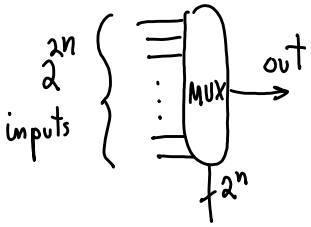
$$= 1/((2/10) + (2^3/10)/2^5)$$

$$= 10/(2 + 1/4) = 40/(8+1) = \frac{40}{9} = 4 + 4/9$$

Other Ideas?

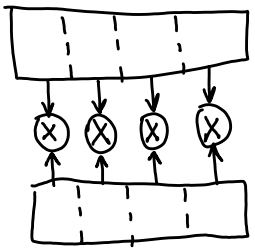


How fast?



→ delay?
area?

Combine



Do mod arith in parallel, combine results
How?

Evaluating Performance

- Performance best determined by running a real application
  - Use programs typical of expected workload
    - e.g., compilers/editors, scientific applications, graphics, etc.
- Microbenchmarks
  - Small programs, synthetic or kernels from larger applications
  - Nice for architects and designers
  - Can be misleading
- Benchmarks
  - Collection of real programs that companies have agreed on
  - Components, programs, inputs & outputs, measurements, rules, metrics
  - Can still be abused

in typical environment?

⇒ Build compiler optimized for benchmark?
⇒ "Buggy" ⇒ skips work?

The SPEC CPU Benchmark Suite
(System Performance Evaluation Cooperative)

gcc →
GO →
QM sim →
XML parse →
speech recog. →

SPEC2006 benchmark description	SPEC2006	Benchmark name by SPEC generation: SPEC2000	SPEC95	SPEC92	SPEC89
GNU C compiler	←	←	←	←	gcc
Interpreted string processing	←	←	perl		espresso
Combinatorial optimization	←	mcf			li
Block-sorting compression	←	bzip2		compress	eqntott
Go game (AI)	go	vortex	go	sc	
Video compression	h264avc	gzip	ijpeg		
Games/path finding	astar	eon	m88ksim		
Search gene sequence	hmmer	twolf			
Quantum computer simulation	libquantum	vortex			
Discrete event simulation library	omnetpp	vpr			
Chess game (AI)	sjeng	crafty			
XML parsing	xalancbmk	parser			
CFD/blast waves	bwaves				fpppp
Numerical relativity	cactusADM				tomcatv
Finite element code	calculix				doduc
Differential equation solver framework	dealII				nasa7
Quantum chemistry	gamess				spice
EM solver (freq/time domain)	GemsFDTD			swim	matrix300
Scalable molecular dynamics (~NAMD)	gromacs		apsi	hydro2d	
Lattice Boltzman method (fluid/air flow)	lbm		mgrid	su2cor	
Large eddie simulation/turbulent CFD	LESlie3d	wupwise	applu	wave5	
Lattice quantum chromodynamics	milc	apply	turb3d		
Molecular dynamics	namd	galgel			
Image ray tracing	povray	mesa			
Spare linear algebra	soplex	art			
Speech recognition	sphinx3	equake			
Quantum chemistry/object oriented	tonto	facerec			
Weather research and forecasting	wrf	ammp			
Magneto hydrodynamics (astrophysics)	zeusmp	lucas			
		fma3d			
		sixtrack			

Other Benchmarks

- Scientific computing: Linpack, SpecOMP, SpecHPC, …
- Embedded benchmarks: EEMBC, Dhrystone, …
- Enterprise computing
  - TCP-C, TPC-W, TPC-H
  - SpecJbb, SpecSFS, SpecMail, Streams,
- Other
  - 3Dmark, ScienceMark, Winstone, iBench, AquaMark, …

- Watch out: your results will be as good as your benchmarks
  - Make sure you know what the benchmark is designed to measure
  - Performance is not the only metric for computing systems
    - Cost, power consumption, reliability, real-time performance, …

Summarizing Performance

- Combining results from multiple programs into 1 benchmark score
  - Sometimes misleading, always controversial…and inevitable
  - We all like quoting a single number

- 3 types of means
  - Arithmetic: for times

$$AM = \frac{1}{n}\sum_{i=1}^{n}\left(Weight_i\right)\cdot Time_i$$

  - Harmonic: for rates

$$HM = \frac{n}{\sum_{i=1}^{n}\frac{\left(Weight_i\right)}{Rate_i}}$$

  - Geometric: for ratios

$$GM = \left(\prod_{i=1}^{n} Ratio_i\right)^{\left(\frac{1}{n}\right)}$$

find ratio $\bar{r}$ s.t.

$r_1 \cdot r_2 \cdots r_n x = (\bar{r})^n x$

$R \Rightarrow (T_{R_1}, 10\,T_{R_1})$

We normalize by using a reference machine R to get the speedups w.r.t. benchmark-1 and benchmark-2 (b-1, b-2).

$S_{A\text{-}R_1} = \frac{T_{R_1}}{100}$ $\quad S_{A\text{-}R_2} = \frac{10\,T_{R_1}}{4}$

$S_{B\text{-}R_1} = \frac{T_{R_1}}{200}$ $\quad S_{B\text{-}R_2} = \frac{10\,T_{R_1}}{1}$

Suppose R's time on b-2 is 10 times its time for b-1. (We can always express R's times in terms of one of its benchmark times, no matter how many b's there are.)

To combine our speedups w.r.t R, let's try getting a mean of these for A, and a mean for B, then taking the ratio of those mean speedups.

$\overline{S}_{A\text{-}R} = \frac{1}{2}\left(\frac{T_{R_1}}{100} + \frac{10\,T_{R_1}}{4}\right)$

$\overline{S}_{B\text{-}R} = \frac{1}{2}\left(\frac{T_{R_1}}{200} + \frac{10\,T_{R_1}}{1}\right)$

$\Rightarrow \frac{\overline{S}_{A\text{-}R}}{\overline{S}_{B\text{-}R}} = \frac{T_{R_1}/2\left(\frac{4+1000}{400}\right)}{T_{R_1}/2\left(\frac{1+2000}{200}\right)} = \left(\frac{1}{2}\right)\left(\frac{1004}{2001}\right) \cong 1/4\ ?$

$\left(\frac{4+100r}{1+200r}\right) \to \frac{1}{2}$ as $r \to \infty$; $\to 4$ as $r \to 0$

This doesn't seem to have worked very well. The effect of R can make all the difference: if R's time on b-2 had been one-thousandth of its b-1 time, the result would have been an overall speedup of 2. Let's try a different mean, the geometric mean.

$\overline{S}_{A\text{-}R} = G(S_{A\text{-}R_1}, S_{A\text{-}R_2}) = \sqrt{\left(\frac{T_{R_1}}{100}\right)\left(\frac{10\,T_{R_1}}{4}\right)} = \sqrt{\frac{10\,T_{R_1}^2}{400}} = \frac{(\sqrt{10}\,T_{R_1})}{2 \cdot 10}$

$\overline{S}_{B\text{-}R} = G(S_{B\text{-}R_1}, S_{B\text{-}R_2}) = \sqrt{\left(\frac{T_{R_1}}{200}\right)\left(\frac{10\,T_{R_1}}{1}\right)} = \sqrt{\frac{10\,T_{R_1}^2}{200}} = \frac{(\sqrt{10}\,T_{R_1})}{\sqrt{2} \cdot 10}$

$\frac{\overline{S}_{A\text{-}R}}{\overline{S}_{B\text{-}R}} = \frac{(\sqrt{10}\,T_{R_1})}{(\sqrt{10}\,T_{R_1})}\left(\frac{T_{B_1} \cdot T_{B_2}}{T_{A_1} \cdot T_{A_2}}\right)^{1/2} \cong 0.7$ $\quad$ [recall $S_{A\text{-}B_1} = 2$, $S_{A\text{-}B_2} = 1/4$]

weights on R's times cancel.

---- (A's point of view): S_avg in [2, 0.25] ===> 30 % slowdown

---- (B's point of view): S_avg in [0.5, 4] ===> 40 % speedup

at least it's consistent

The real world? Suppose job mix = (**n1** runs of b-1) + (**n2** runs of b-2)

$S_{A\text{-}B} = \frac{n_1 T_{B_1} + n_2 T_{B_2}}{n_1 T_{A_1} + n_2 T_{A_2}} = \frac{200 n_1 + n_2}{100 n_1 + 4 n_2} \cong \frac{200 + a}{100 + 4a}$ $\quad (a = n_2/n_1)$

$\to 1/4$, $a \to \infty$; $\to 2$, $a \to 0$

Sanity check: Given our result above, what **a** does GM appear to assume?

$$\frac{200+a}{(100+4a)} \approx 3/4 \Rightarrow (200+a)4 = (100+4a)3 \Rightarrow 500 = 8a$$

$$a \cong 62$$

$$\eta_2 = 62\ \eta_1$$

For every short job (b1) we do 62 long jobs (b2)?

What if we hadn't taken the SQRT in GM?

$$\bar{S}_{A-B} = \frac{1}{2} \Rightarrow (200+a)2 = (100+4a) \Rightarrow a = 50$$

$$HM = \frac{1}{\sum r_i} \quad \text{where } r_1 = \frac{W_1}{T_1},\ r_2 = \frac{W_2}{T_2} \cdots \quad [r_i = q_i]$$

[find $\bar{r}$ s.t., if we did all work at same rate, takes same time]

$$\frac{(W_1 + W_2 + \cdots W_n)}{\bar{r}} = (T_1 + T_2 + \cdots T_n)$$

$$\bar{r} = \frac{W}{\left(\frac{W_1}{r_1}\right) + \left(\frac{W_2}{r_2}\right) \cdots \left(\frac{W_n}{r_n}\right)}$$

$$= 1 \Big/ \sum \frac{(W_i/W)}{r_i} \qquad (W_i/W) = \omega_i$$

$$W = \sum W_i$$

We have reference Times:

$$T_{R_1} = \frac{W_1}{q_{R-1}} \qquad T_{R_2} = W_2/q_{R-2} \qquad [\text{assume } q_{R-1} = q_{R-2} = q_R]$$

$$W_1 = T_{R-1} q_R \qquad W_2 = T_{R-2} q_R$$

Use that to get weights:

$$\omega_i = W_i / \sum W_i$$

$$\omega_1 = \frac{W_1}{W_1+W_2} = \frac{T_{R_1} q_R}{(T_{R_1} q_R + T_{R_2} q_R)} = \frac{T_{R_1}}{T_{R_1}+T_{R_2}}$$

$$= 1/(1+10) = 1/11$$

$$\omega_2 = \frac{T_{R_2}}{T_{R_1}+T_{R_2}} = \frac{10}{1+10} = 10/11$$

$$\bar{q}_A = \frac{1}{\sum \omega_i / q_{A-i}}$$

Given our assumption that $q_{R-1} = q_{R-2} = q_R$

$$\frac{W_1 = T_{R-1}\, q_R}{W_2 = T_{R-2}\, q_R} = \frac{T_{R-1}}{T_{R-2}} = \frac{T_{R-1}}{10\, T_{R-1}} = 1/10$$

or $W_2 = 10\, W_1$

$$q_{A-1} = \frac{W_1}{T_{A-1}} = \frac{W_1}{100}$$

$$q_{A-2} = \frac{W_2}{T_{A-2}} = \frac{10\, W_1}{4}$$

$$\bar{q}_A = \frac{1}{\left(\frac{(1/11)}{W_1/100} + \frac{(10/11)}{10\, W_1/4} \right)} = \frac{(11)}{\frac{100}{W_1} + \frac{10 \cdot 4}{10\, W_1}} = \frac{11\, W_1}{(100+4)}$$

(T_{A-1}, T_{A-2})

$$\bar{q}_B = \frac{11\, W_1}{(200+1)}$$

$$\rightarrow \quad S^{HM}_{A-B} = \bar{q}_A / \bar{q}_B = \frac{(200+1)}{(100+4)} \cong 2$$

Suppose, again, n_1 runs of b-1 and n_2 runs of b_2, w/ $a = n_2/n_1$.
Comparing our "real world" performance, what a does HM imply?

$$S^{real}_{A-B} = \left(\frac{200 + a}{100 + 4a} \right) = S^{HM}_{A-B} = \frac{2001}{104}$$

$$\Rightarrow \quad 104(200 + a) = 201(100 + 4a)$$

$$\Rightarrow \quad 20800 + 104a = 20100 + 804a$$

$$\Rightarrow \quad 700 = 700a$$

$$\boxed{a = 1}$$

Principles of Computer Design

- Take Advantage of Parallelism
  - e.g. multiple processors, disks, memory banks, pipelining, multiple functional units
- Principle of Locality
  - Reuse of data and instructions
- Focus on the Common Case
  - Amdahl's Law

$$\text{Execution time}_{\text{new}} = \text{Execution time}_{\text{old}} \times \left((1 - \text{Fraction}_{\text{enhanced}}) + \frac{\text{Fraction}_{\text{enhanced}}}{\text{Speedup}_{\text{enhanced}}} \right)$$

1.02 #bytes per frame, time per file (cache, DRAM, ...)
1.03 avg CPI, CR, performance
1.04-05 CPI by class, CR, instr. mix,
1.06 compilers, avg CPI, CR, speedup, CPI by class, peak performance versus
1.07 Voltage scaling laws, C, power, GM, %change,
1.08 dynamic power, C, V
1.09 static and dynamic power, voltage dependence
1.10 multi-cores, #instructions, CPIs, execution time, power
1.11 die yield and cost
1.12 SPEC ratio from times
1.13 Faster clock, change ISA ==> fewer instructions executed, CPI vs CR
1.14 Performance measured by MFLOPS or MIPS versus overall
1.15 Amdahl's Law (improving only a fraction)
1.16 Speedup w/ communication costs