Elements of Algorithms

Mark Maloof
Department of Computer Science
Washington, DC 20057

September 29, 2015

Four Critical Elements

- Algorithms have some subset of the following critical elements:
  1. simple statements, including but not limited to:
     - input statements
     - assignment statements
     - output statements
     - return statements
  2. sequences of statements, which are also statements
  3. branching statements
  4. looping statements

Algorithm for Simple Interest

1: **input** r, b, and m ▷ input statement
2: $i \leftarrow r \cdot b \cdot m$ ▷ assignment statement
3: **output** i ▷ output statement

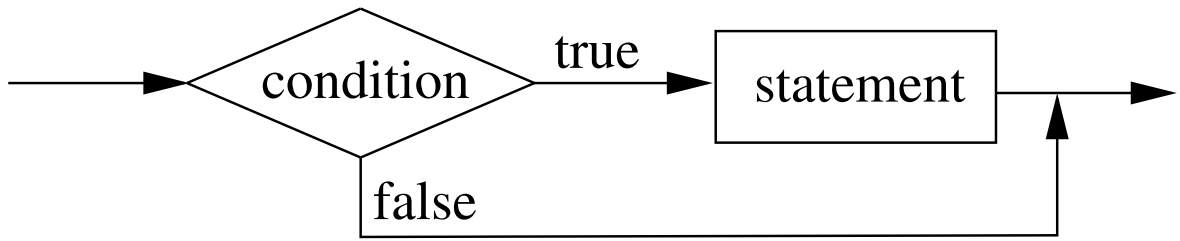
Another Algorithm for Simple Interest

1: **input** r ▷ input statement
2: **input** b ▷ input statement
3: **input** m ▷ input statement
4: $i \leftarrow r$ ▷ assignment statement
5: $i \leftarrow i \cdot b$ ▷ assignment statement
6: $i \leftarrow i \cdot m$ ▷ assignment statement
7: **output** i ▷ output statement

Branching: If-then Statement

if some condition is true **then**
    statement (or sequence)
end if

Flowchart for an if-then Statement



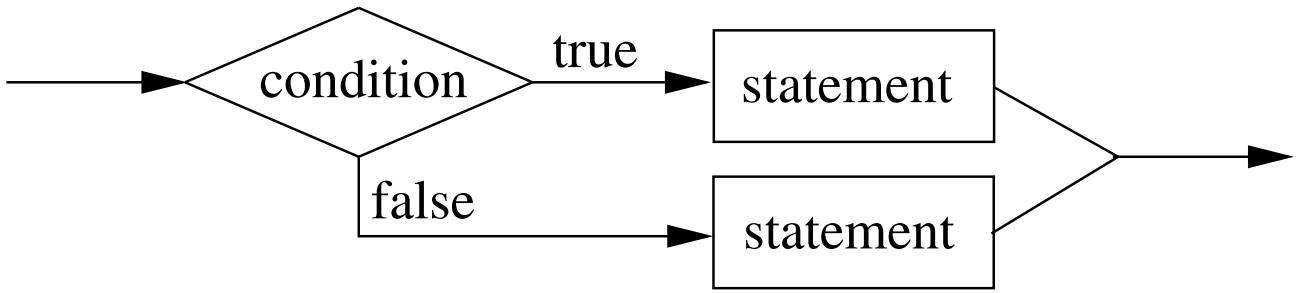
Example of an if-then Statement

```
1: input grade
2: if grade > 64 then
3:     output pass
4: end if
5: if grade ≤ 64 then
6:     output fail
7: end if
```

Branching: If-then-else Statement

if some condition is true **then**
    statement (or sequence)
else
    statement (or sequence)
end if

Flowchart for an if-then-else Statement



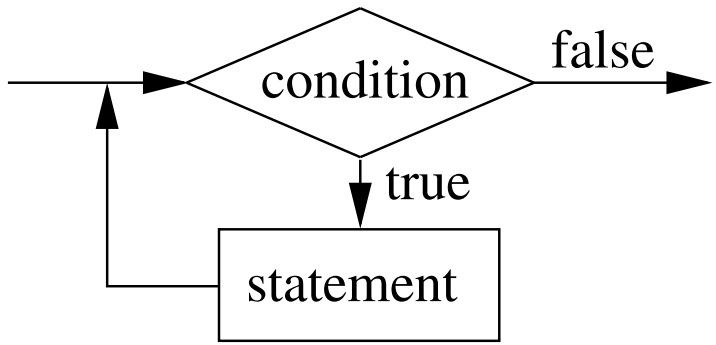
Example of an if-then-else Statement

```
1: input grade
2: if grade > 64 then
3:     output pass
4: else
5:     output fail
6: end if
```

Looping: While Statement, While Loop

while some condition is true **do**
    statement (or sequence)
end while

Flowchart for a While Loop



Example of a While Loop

```
1: input grade
2: while there is a grade do
3:     if grade > 64 then
4:         output pass
5:     else
6:         output fail
7:     end if
8:     input grade
9: end while
```

Repeat-until Loop

repeat
    statement (or sequence)
until some condition is true

Equivalent to:

statement (or sequence)
while some condition is false **do**
    statement (or sequence)
end while

For Loop

for $i \leftarrow b, e$ **do**
    statement (or sequence)
end for

Equivalent to:

$i \leftarrow b$
while $i \leq e$ **do**
    statement (or sequence)
    $i \leftarrow i + 1$
end while

For-each Loop

for each element of some collection **do**
    statement (or sequence)
end for

Equivalent to:

$i \leftarrow 1$
$e \leftarrow$ the number of elements in the collection
while $i \leq e$ **do**
element $\leftarrow i$th element of the collection
    statement (or sequence)
    $i \leftarrow i + 1$
end while

Example of a For-each Loop

```
1: Let Grades be a sequence or list of grades
2: input Grades
3: for each grade in Grades do
4:     if grade > 64 then
5:         output pass
6:     else
7:         output fail
8:     end if
9: end for
```

Algorithm for Binary-to-Decimal Conversion

1: Let D be a decimal number, set to zero
2: Let B be a binary number, set to zero
3: **input** B
4: Let B' be B with its digits reversed
5: $i \leftarrow 0$
6: **for each** binary digit $b \in B'$ **do**
7: $\quad D \leftarrow D + b \cdot 2^i$
8: $\quad i \leftarrow i + 1$
9: **end for**
10: **output** D

Program for B2D Conversion in Julia

```
D = 0
B = {1,1,0,0,1}
BPrime = B[end:-1:1]
i = 0
for b in BPrime
  D = D + b * 2^i
  i = i + 1
end
println( D )
```

Program for B2D Conversion in C

```
#include <stdio.h>
#include <math.h>

int main()
{
  int b[] = { 1, 0, 0, 1, 1 };
  int n = 5;
  int d = 0;
  int i = 0;
  for ( i = n - 1; i >= 0; i = i - 1 ) {
    d = d + b[i] * (int) pow( 2.0, i );
  }
  printf( "%d\n", d );
  return 0;
}
```

Algorithm for Decimal-to-Binary Conversion

1: Let B be an empty sequence of binary digits
2: Let D be a decimal number, set to zero
3: **input** D
4: **while** $D \neq 0$ **do**
5: $\quad r \leftarrow D \mod 2$
6: $\quad$ Add r as the left-most digit of B
7: $\quad D \leftarrow D \div 2$ (integer division)
8: **end while**
9: **output** B

Program for D2B Conversion in Julia

```
B = {}
D = 25
while D > 0
  r = D % 2
  unshift!( B, r )
  D = div( D, 2 )
end
println( B )
```

Program for D2B Conversion in C

```
#include <stdio.h>
#include <math.h>

int main()
{
  int d = 25;
  int n = (int) ceil( log2( d ) );
  int b[n];
  int i = n - 1;
  int r = 0;
  while ( d > 0 ) {
    r = d % 2;
    b[i] = r;
    i = i - 1;
    d = d / 2;
  }
  for ( i = 0; i < n; i = i + 1 ) {
    printf( "%d", b[i] );
  }
  printf( "\n" );
  return 0;
}
```