Note Title 1/21/2015

Sets

Def^n: Set S is a collection of objects

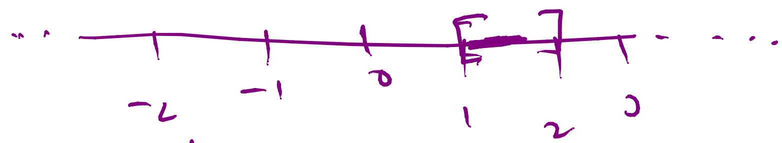
Eg: $S = \{1, \text{chair}, \text{"Math"}, \ldots\}$

$N = \{0, 1, 2, \ldots\}$...

$Z = \{\ldots -2, -1, 0, 1, 2, \ldots\}$

$Q = \{ p/q \mid p \in Z,\ q \in Z,\ q \neq 0 \}$

$R =$ set of all real numbers

$[1,2] =$ 

$$[1,2] = \{ x \in \mathbb{R} \mid 1 \le x \le 2 \}$$

$$[a,b] = \{ x \in \mathbb{R} \mid a \le x \le b \}$$

$$(1,4] = \{ x \in \mathbb{R} \mid 1 < x \le 4 \}$$

$$(a,b) = \{ x \in \mathbb{R} \mid a < x < b \}$$

DEF^n:

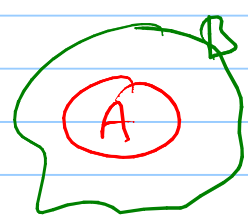
$\cup$, $\cap$, $\subseteq$, $\subset$, Compliment

union, intersection, subset, strict subset

Eg: Let $A = \{1, 3, 5\}$ $B = \{1, 2, 3, 7\}$

$A \cup B = \{1, 2, 3, 5, 7\}$; $A \cap B = \{1, 3\}$

Subset:



$A \subseteq B$

$A \subset B$

Def^n: Let A, B be two sets

$$A \subseteq B \iff (\forall x)\left[x \in A \Rightarrow x \in B\right]$$

Def^n: $A \not\subseteq B$

$$\sim(A \subseteq B) \iff \sim\left\{(\forall x)\left[(x \in A) \Rightarrow x \in B\right]\right\}$$

$$A \not\subseteq B \iff (\exists x) \sim \left[(\underline{x \in A}) \Rightarrow (\underline{x \in B})\right] \qquad p \Rightarrow q$$

$$\equiv$$

$$A \not\subseteq B \iff (\exists x) \sim \left[\sim(x \in A) \vee (x \in B)\right] \qquad \sim p \vee q$$

$$A \not\subseteq B \iff (\exists x) \left[\sim\sim(x \in A) \wedge \sim(x \in B)\right] \rightarrow \sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$A \not\subseteq B \iff (\exists x)\left[(x \in A) \wedge (x \notin B)\right]$$

$\otimes$

DEF^n: Let A, B be two sets

$(A = B) \Leftrightarrow (A \subseteq B) \wedge (B \subseteq A)$

Complement Eg: $A = \{1, 5, 7\}$ $X = \mathbb{N}$

$A^c = \{0, 2, 3, 4, 6, 8, 9, 10, \ldots\}$

Eg: $A = \{1, 2, 3\}$ $B = \{2, 3, 4, 5\}$

$A - B = \{1\}$; $B - A = \{4, 5\}$

Notation: ϕ empty set. (not: $S = \{\ \}$)

Ex: $S = \{ x \in \mathbb{N} \mid x = \sqrt{2} \} = \{\ \} = \phi$

$S = \{ x \in \mathbb{Q} \mid x = p/q \ ; \ q = 0 \}$

? $\phi \subseteq \mathbb{N}$

$(\forall x) [x \in \phi \implies x \in \mathbb{N}]$

True Vacuous Proof.

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<u>Eg:</u>

$A = \{1,2,3\}$, $B = \{a,b\}$

$A \times B = \{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)\}$

$B \times A = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3)\}$

$\times$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

? $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

<u>Sol^n:</u> must show $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ — (1)

$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ — (2)

1. Let $x \in A \cup (B \cap C) \Rightarrow (x \in A)$ or $x \in (B \cap C)$

$x \in (A \cup B)$ and $x \in (A \cup C)$ $\qquad$ $(x \in B)$ and $(x \in C)$

$x \in (A \cup B) \cap (A \cup C)$ $\qquad$ $x \in (A \cup B)$ and $x \in (A \cup C)$

$x \in (A \cup B) \cap (A \cup C)$

$\therefore\ x \in A \cup (B \cap C) \Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\therefore\ A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

2. $x \in (A \cup B) \cap (A \cup C) \Rightarrow x \in (A \cup B)$ AND $x \in (A \cup C)$

$(x \in A$ or $x \in B)$ $\wedge$ $(x \in A)$ or $(x \in C)$

Supp $x \in A \implies x \in A \cup (B \cap C)$

Supp $x \notin A \implies x \in B \wedge x \in C$

$\implies x \in (B \cap C)$

$\implies x \in A \cup (B \cap C)$

$\therefore (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

$\therefore (A \cup B) \cap (A \cup C) = A \cup (B \cap C)$

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$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Eg: $A = \{1, 2\}$ $\quad$ $B = \{1, 2, 3\}$

$$A \subseteq B$$

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Propn: $\sqrt{3}$ is irrational.

Lemma: Let $a \in \mathbb{N}$ Then if $\underbrace{3 \text{ divides } a^2}_{P}$ then $\underline{3 \text{ divides}}\ a$

Proof: Let $a \in \mathbb{N}$ and 3 divides a^2

assume 3 does not divide a

$a = 3q + r$; $r = 1, 2$

$\Longrightarrow a^2 = (3q+r)^2 = 9q^2 + 6qr + r^2$

$a^2 - 9q^2 - 6qr = r^2$

$\Longrightarrow 3 \mid r^2$; contradiction

here $3 \mid a$

Proof: Assume $\sqrt{3}$ rational.

$\Longrightarrow \sqrt{3} = p/q$; $q \neq 0$ & $\gcd(p,q) = 1$.

$\sqrt{3}\, q = p$; $3q^2 = p^2$.

$\Longrightarrow 3 \mid p^2 \Longrightarrow 3 \mid p$ (Lemma)

$\Rightarrow \quad p = 3a$

$3q^2 = (3a)^2 \Rightarrow \quad 3q^2 = 9a^2$

$\Rightarrow \quad q^2 = 3a^2$

$\Rightarrow \quad 3 \mid q^2 \Rightarrow 3 \mid q$

$\therefore \quad \gcd(p, q) \geq 3$

$\therefore \quad \sqrt{3}$ irrational

$\checkmark$

Prob: n is odd $\Longleftrightarrow$ $(5n+6)$ is odd

$$(p \Longleftrightarrow q) \equiv (p \to q) \wedge (q \to p)$$

Proof:

$\Rightarrow$ Assume n is odd

$\Rightarrow$ $n = 2k+1 \implies 5n+6 = 5(2k+1)+6$

$5n+6 = 10k+5+6 = 10k+10+1$

$5n+6 = 2(5k+5)+1$

$\therefore$ $(5n+6)$ is odd

$\Leftarrow$ assm $(5n+6)$ is odd

$\Longrightarrow$ $5n$ is odd

But if n is even $5n$ is even

$\therefore$ n is odd

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