Note Title 1/14/2015

<u>Prop^n:</u> (Last Thm)

$(n \in \mathbb{Z})\ [\ n \text{ even} \implies n^2 \text{ even}\ ]$

Direct Proof.

<u>Prop^n:</u> $(\forall n \in \mathbb{Z})\ [\ \underbrace{n^2 \text{ even}}_{p} \implies \underbrace{n \text{ even}}_{q}\ ]$

Proof: [This will not work

Let $(n \in \mathbb{Z})$ a even

$\implies (\exists m \in \mathbb{Z})$ such that $n^2 = 2m$

$\Rightarrow \quad n = \sqrt{2m}$)

$p \Rightarrow q \equiv \sim q \Rightarrow \sim p \equiv \sim p \vee q$

Proof: by Contraposition. ($\sim q \rightarrow \sim p$)

i.e must show (n odd $\rightarrow n^2$ odd)

Let $n \in \mathbb{Z}$ and n odd.

$\Rightarrow \quad \exists m \in \mathbb{Z}$ s.t $\quad n = 2m+1$

$\Rightarrow \quad n^2 = (2m+1)^2 = 4m^2+4m+1$

$\Rightarrow \quad n^2 = 2(2m^2+2m)+1$

$\therefore$ [n odd $\Longrightarrow$ n^2 odd]

[$\sim q \longrightarrow \sim p$]

$\therefore$ [n^2 even $\Longrightarrow$ n even]

X

<u>DEF^n:</u> q is a rational number if and only if $\exists\, n, m \in \mathbb{Z}$ such that $m \neq 0$ and

$q = n/m$ $\quad (\gcd(n,m) = 1)$

Prop^n: $\sqrt{2}$ is an Irrational number.

p

$\boxed{\sim p \Rightarrow F} \equiv \sim\sim p \vee F = p \vee F = p$

assume $\sqrt{2}$ is rational.

$\Rightarrow \sqrt{2} = \dfrac{n}{m}$; where $n, m \in \mathbb{Z}$, $m \neq 0$ $\wedge$ $\gcd(n,m) = 1$

$\Rightarrow m \cdot \sqrt{2} = n \Rightarrow (m \cdot \sqrt{2})^2 = n^2$

$\Rightarrow m^2 \cdot 2 = n^2$ —— (1)

$\therefore$ n^2 is even

by the previous result n is even

$\Longrightarrow$ $n = 2 \cdot k$ (where $k \in \mathbb{Z}$)

$\Longrightarrow$ $n^2 = 4k^2$

$\Longrightarrow$ $2m^2 = 4k^2$ $\Longrightarrow$ $m^2 = 2k^2$

$\Longrightarrow$ m^2 even $\Longrightarrow$ m even

$\Longrightarrow$ 2 is a factor of n and m

contradiction to $\gcd(n,m) = 1$

$\therefore \underline{\sim p \Rightarrow F}$

$\Rightarrow p$

$\therefore \sqrt{2}$ is irrational

$\times$

Prop^n: $\exists$ two irrational numbers r, s such that (r^s) is rational.

Sol^n: Let $s = \sqrt{2}$, $r = \sqrt{2}$

Consider $r^s = \left(\sqrt{2}^{\sqrt{2}}\right)$

Case (1) : r^s is rational

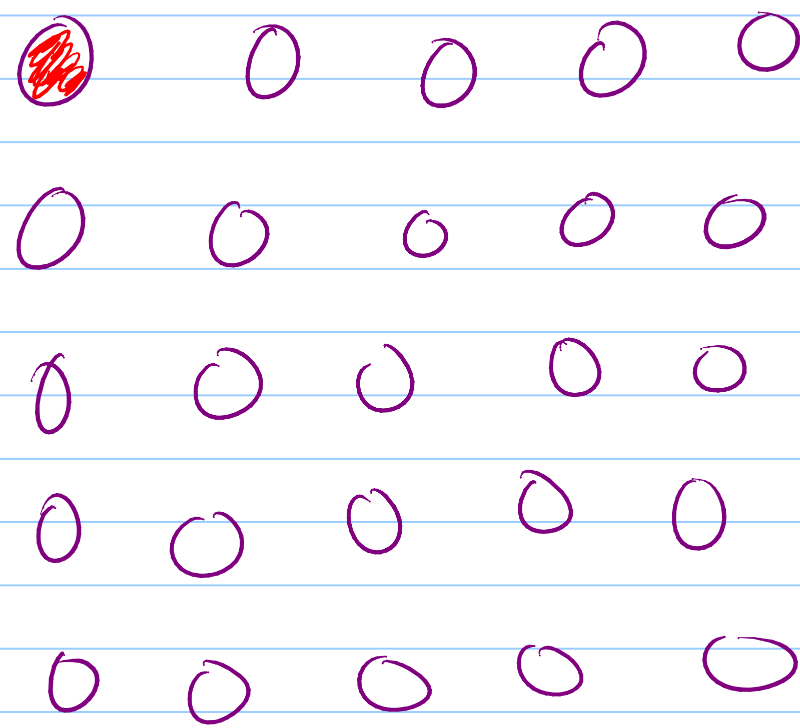
done!.

Case (2) : $\left(\sqrt{2}^{\sqrt{2}}\right)$ irrational.

Consider $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \left(\sqrt{2}\right)^{\sqrt{2}\cdot\sqrt{2}} = \left(\sqrt{2}\right)^2 = 2$

Pase 99

Champ



x

Eg: $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})\,[\underbrace{x + y}_{P(x,y)}] = 0$

$\equiv (\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})\; P(x,y)$

$\equiv T$

$\sim[(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})\; P(x,y)]$

$(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})\,[x + y = 0]$

F

$\sim(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})\,[x + y = 0]$

$(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})\,[x + y \neq 0]$

$$(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z}) \sim P(x,y)$$

$$(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})\ [x + y \neq 0]$$

$(\forall x)(\forall y)\ P(x,y)$	$\sim(\forall x)(\forall y)\ P(x,y)$ $\equiv (\exists x)(\exists y) \sim P(x,y)$
$(\exists x)(\exists y)\ P(x,y)$	$\sim(\exists x)(\exists y)\ P(x,y)$ $\equiv (\forall x)(\forall y) \sim P(x,y)$

Chap 2 (SETS)

(1) **DEF^n:** A set S is a collection of objects.

Eg: $S = \{1, \text{pen}, \text{computer}, \text{"make"}\}$