Note Title 1/12/2015

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(1) $\underline{\underline{(-4) > 2}} \Longrightarrow (-4)^2 > (2)^2$

$(-4) > 2 \Longrightarrow 16 > 4$

$\parallel \qquad\qquad =$

F $\qquad\qquad$ T

(2) $\left(\sqrt{2} > \frac{3}{2}\right) \Longrightarrow (\sqrt{2})^2 > \left(\frac{3}{2}\right)^2 \Longrightarrow 2 > \frac{9}{4}$

F $\qquad\qquad$ F

(1) <u>Operator Precedence</u>

(1) $\sim$

(2) $\wedge$

(3) $\vee$

(4) $\longrightarrow$

<u>DE'Morgan's Laws</u>

(1) $\sim(p \wedge q) \equiv \sim p \vee \sim q$ $\longleftarrow$ ✓

(2) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

$$\sim(x \vee y) \equiv \sim x \wedge \sim y$$

Let $\sim p \equiv x \implies p = \sim x$

$\sim q \equiv y \implies q = \sim y$

$$\sim(\sim p \vee \sim q) \equiv p \wedge q$$

$$(\sim p \vee \sim q) \equiv \sim(p \wedge q)$$

(1) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(2) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(3) p (This mean p is true)

$p \rightarrow q$ (" " $p \rightarrow q$ is true)

$\therefore q$ (you can conclude q is true)

(1) <u>Universal Quantifier</u>

eg: $P(x) \equiv x+1 \geq 0$ (where x is an integer ; $x \in \mathbb{Z}$)

1

$P(1) \equiv 1+1=2 \geq 0$ T

$P(0) \equiv 0+1=1 \geq 0$ T

$P(-5) \equiv$ F

$$(\forall x \in \mathbb{Z})[P(x)] \equiv (\forall x \in \mathbb{Z})[x+1 \geq 0]$$

2

$$P(x) \equiv x^2+1 \geq 0$$

$$(\forall x \in \mathbb{Z})\, P(x) \equiv (\forall x)\left[x^2+1 \geq 0\right]$$

3 $(\forall x \in \mathbb{Z})\, p(x) \equiv (\forall x)\left[x^2+1 > 1\right]$

Existential Quantifier $\exists$

Eg: $(\exists x \in \mathbb{Z})\left[x+5 = 0\right]$

Observation:

- $\sim\left[(\forall x)\ P(x)\right] \equiv (\exists x) \sim P(x)$

- $\sim\left[\exists x\ P(x)\right] \equiv (\forall x) \sim P(x)$

DEF^n: Let n be an integer; n is said to be even if $\exists$ an integer m such that

$$n = 2m$$

<u>DEF^n:</u> Let n be an integer; n is said to be <u>odd</u> if $\exists$ an integer m such that

$$n = 2m+1$$

<u>Proposition:</u> Let n be an even integer then n^2 is even.

$$\left[\begin{array}{l} P(n):\ n \text{ is even};\\ Q(n):\ n^2 \text{ is even} \end{array}\right\} \quad (\forall n \in \mathbb{Z})\left[P(n) \to Q(n)\right]$$

Sol^n: Let n be any even integer

$\Rightarrow \exists$ an $m \in \mathbb{Z}$ such that $n = 2m$

$\Rightarrow n^2 = (2m)^2 = 4m^2 = 2\cdot(2m^2)$

$\therefore n^2$ is even