Propositional Logic

Note Title 1/7/2015

Const: T, F

DEF^n: Proposition is a sentence that is either T or F but not both

NOTATION Let, p, q, r, s... denote Propositional variables.

DEF^n: (operation) NOT, OR, AND, implication

Notation

$\neg$	$\sim$	
$\vee$	$\vee$	
$\wedge$	$\wedge$	
$\rightarrow$	$\rightarrow$	

NOT

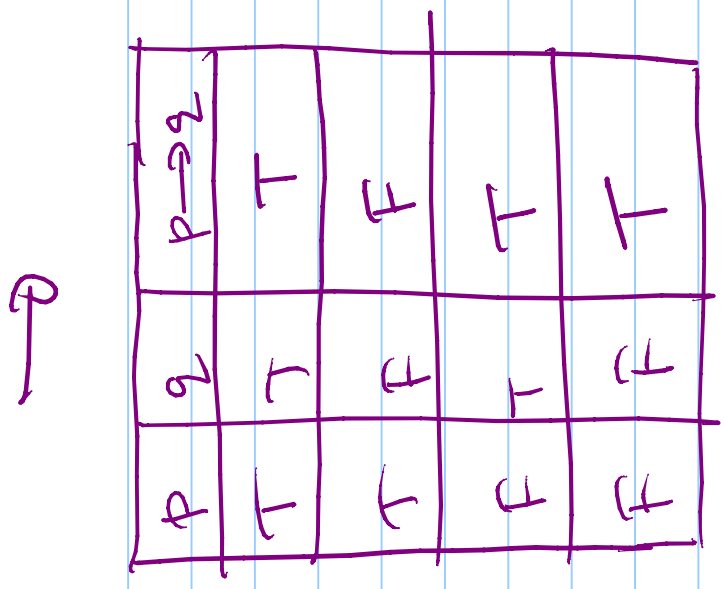
P	$\sim P$
T	F
F	T

OR

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

AND

$P \wedge Q$
T
F
F
F


p
p⇒q
T
T
T
T
q
T
T
T
T
p
T
T
T
T

Obs: $p \to q \neq q \to p$

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Obs $p \to q \# \sim p \Longrightarrow \sim q$

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$
T	T	T	T	T	F
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

RESULT: $p \rightarrow q \equiv \sim p \vee q$

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\oplus$ xor (exclusive or)

p	q	$p \oplus q$	$p \leftrightarrow q$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	T

$(p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

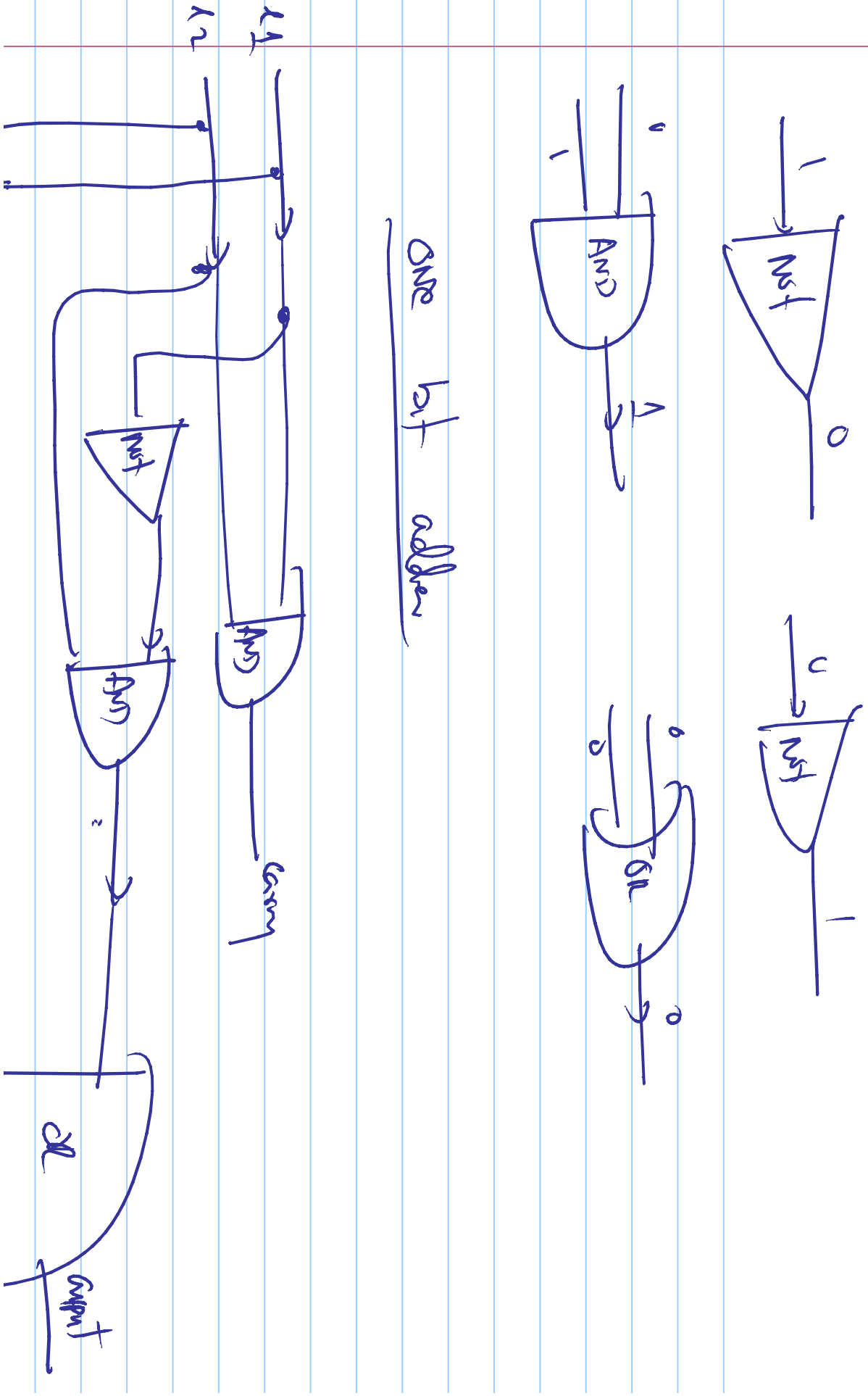
$p \oplus q \equiv (\neg p \wedge q) \vee (p \wedge \neg q)$

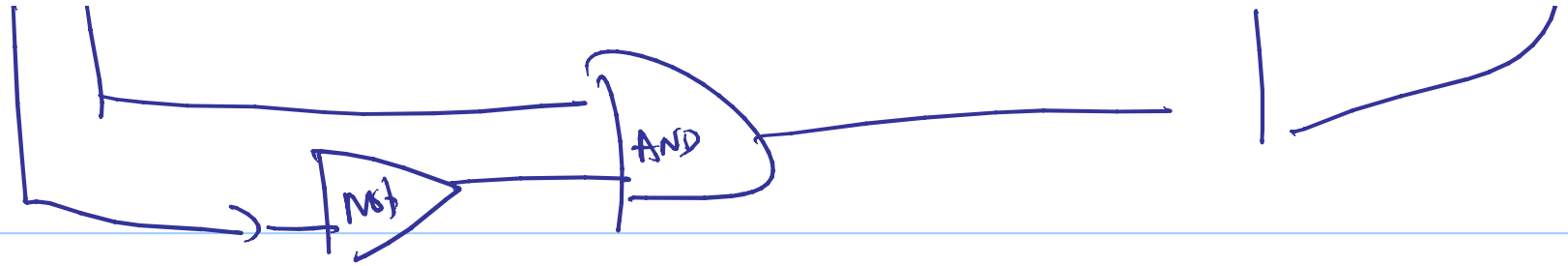
Let $T \equiv 1$, $F \equiv 0$

input 1	input 2	output	carry
0	0	0	0
1	0	1	0
0	1	1	0
1	1	0	1

$$\text{carry} \equiv (\text{input} \wedge \text{input 2})$$

$$\text{output} \equiv (\sim \text{input 1} \wedge \text{input 2}) \vee (\text{input 1} \wedge \sim \text{input 2})$$


AND
OR
NOT



input →				out	
I1	I2	carry		output	carry
0	0	0		0	0
0	1	0		1	0
1	0	0		1	0
1	1	0		0	1
0	0	1		1	0
0	1	1		0	1
1	0	1		0	1
1	1	1		1	1