HW 5 sol   Pase 154

Tuesday, January 27, 2015   9:20 AM

#48) $(\forall x \in \mathbb{R}) \left[\lceil x \rceil - \lfloor x \rfloor = 1 \quad x \notin \mathbb{N} \right]$

$\left[\lceil x \rceil - \lfloor x \rfloor = 0 \quad x \in \mathbb{N} \right]$

Sol^n: $x \in \mathbb{R} \wedge x \notin \mathbb{N}$

Then $x = n + r \quad 0 < r < 1$

$\Rightarrow \lfloor x \rfloor = n \wedge \lceil x \rceil = n+1$

hence $\lceil x \rceil - \lfloor x \rfloor = 1$

also if $x \in \mathbb{N}$

$\lfloor x \rfloor = \lceil x \rceil = x$

$\therefore \lceil x \rceil - \lfloor x \rfloor = 0$

$\otimes$

#52) Let $x \in \mathbb{R} \wedge n \in \mathbb{N}$ such that $x \leq n$

(a) $\Rightarrow$ assume $x \leq n$ [we must show $\lceil x \rceil \leq n$]

Let $\lceil x \rceil = m$ then by def^n

$m - 1 < x \leq m$ ——— (1)

If $m > n \Rightarrow m - 1 \geq n$ ——— (2)

using ① & 2 $n < x$

contradiction to $x \le n$.

hence $m \le n$ i.e $\lceil x \rceil \le n$

$\Leftarrow$ suppose $\lceil x \rceil \le n$ [show $x \le n$]

but $x \le \lceil x \rceil \le n$

$\therefore$ $x \le n$.

⊗

(b) $\Rightarrow$ $n \le x$ [must show $n \le \lfloor x \rfloor$]

Let $\lfloor x \rfloor = m$ $\Rightarrow$ $m \le x < m+1$

suppose $n > m$ $\Rightarrow$ $n \ge m+1 > x$

$\Rightarrow$ $n > x$

contradiction $\therefore$ $n \le m$

i.e $n \le \lfloor x \rfloor$

$\Leftarrow$ suppose $n \le \lfloor x \rfloor$ [must show $n \le x$]

$n \le \lfloor x \rfloor \le x$

⊗

$\boxed{\#54}$ $(\forall\, x \in \mathbb{R})$

① $\lfloor -x \rfloor = -\lceil x \rceil$

Let $\lceil x \rceil = m$

$\Rightarrow m-1 < x \le m$

$\Rightarrow -m+1 > -x \ge -m$

$\Rightarrow -m \le -x < -m+1$

$\Rightarrow \lfloor -x \rfloor = -m$

$\therefore \lfloor -x \rfloor = -\lceil x \rceil$

② $\lceil -x \rceil = -\lfloor x \rfloor$

Let $\lfloor x \rfloor = m$

$\Rightarrow m \le x < m+1$

$\Rightarrow -m \ge -x > -m-1$

$\Rightarrow -m-1 < -x \le -m$

$\Rightarrow \lceil -x \rceil = -m$

$\therefore \lceil -x \rceil = -\lfloor x \rfloor$

$\times$

$\boxed{\#76}$ Let $(x \in \mathbb{R})$ Then

$$\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + 1/3 \rfloor + \lfloor x + 2/3 \rfloor$$

Proof! Let $x \in \mathbb{R} \Rightarrow x = n + r$; where $n \in \mathbb{Z}$ and $0 \le r < 1$

Case (1) $0 \le r < \frac{1}{3}$; $3r < 1$

LHS: $\lfloor 3x \rfloor = \lfloor 3n + 3r \rfloor = 3n + \lfloor 3r \rfloor = 3n.$

RHS: $\lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + 2/3 \rfloor = \lfloor n + r \rfloor + \lfloor n + r + 1/3 \rfloor + \lfloor n + r + \frac{2}{3} \rfloor$

$= n + n + n = 3n$ $\quad (r + 2/3 < \frac{1}{3} + 2/3)$

Case 2: $\frac{1}{3} \le r < \frac{2}{3}$; $\Longrightarrow$ $1 \le 3r < 2$

LHS: $\lfloor 3x \rfloor = \lfloor 3n+3r \rfloor = 3n + \lfloor 3r \rfloor = 3n+1$

since $3r < 2$

RHS: $\lfloor x \rfloor + \lfloor x+\frac{1}{3} \rfloor + \lfloor x+\frac{2}{3} \rfloor =$

$\lfloor n+r \rfloor + \lfloor n+r+\frac{1}{3} \rfloor + \lfloor n+r+\frac{2}{3} \rfloor$

$= n + n + \lfloor r+\frac{1}{3} \rfloor + n + \lfloor r+\frac{2}{3} \rfloor$

$= 2n + \overset{\shortparallel}{0} + n + \overset{\shortparallel}{1} = 3n+1$

$\left[\; r+\frac{1}{3} \le \frac{2}{3}+\frac{1}{3} = 1 \quad ; \quad r+\frac{2}{3} \ge \frac{2}{3}+\frac{2}{3} < 2 \;\right]$

Case (3) $\frac{2}{3} \le r < 1$; $\Longrightarrow$ $2 \le 3r < 3$

LHS: $\lfloor 3x \rfloor = \lfloor 3n+3r \rfloor = 3n + \lfloor 3r \rfloor = 3n+2$

RHS: $\lfloor x \rfloor + \lfloor x+\frac{1}{3} \rfloor + \lfloor x+\frac{2}{3} \rfloor$

$= \lfloor n+r \rfloor + \lfloor n+r+\frac{1}{3} \rfloor + \lfloor n+r+\frac{2}{3} \rfloor$

$= n + \lfloor r \rfloor + n + \lfloor r+\frac{1}{3} \rfloor + n + \lfloor r+\frac{2}{3} \rfloor$

$= 3n + \overset{\shortparallel}{0} + \overset{\shortparallel}{1} + \overset{\shortparallel}{1}$

(Since; $\frac{2}{3}+\frac{1}{3} \le r+\frac{1}{3} < 1+\frac{1}{3}$; $\frac{2}{3}+\frac{2}{3} \le r+\frac{2}{3} < 1+\frac{2}{3}$)

LHS $:= 3n+2$