Page 108 #9, 30, 34, 38

#9) The Text book has the following solution:

Consider: $(100)^2 = 10000$
$(101)^2 = 10201$

so integers : 10001, 10002, ... 10200

cannot be perfect squares of any integer.
There are 200 such integer.

But $(50)^2 = 2500$
$(51)^2 = 2601$

so the numbers 2501, 2502, ... 2600

cannot be perfect square of any numbers.
There are exactly 100 such number.

⊗

#30) $2x^2 + 5y^2 = 14$; show no integer solutions.

sol^n: $2x^2 = 14 - 5y^2$

Since $2x^2 \geq 0 \Rightarrow 14 - 5y^2 \geq 0$

$\Rightarrow 5y^2 \leq 14$

$\Rightarrow y^2 \leq 2.8$

$\Rightarrow -\sqrt{2.8} \le y \le \sqrt{2.8}$

$\Rightarrow -1.67 \le y \le 1.67$

since y is an integer

$-1 \le y \le 1$

So the only values y can take is $-1, 0, 1$

Now $2x^2 = 14 - 5y^2$

$y = -1, 1$ $\quad 2x^2 = 14 - 5 = 9$

$x^2 = 9/2$

$x = \sqrt{9/2}$ not an integer

$y = 0$; $2x^2 = 14$

$x = \sqrt{7}$ not an integer

$\otimes$

(#34) Show $(2)^{1/3}$ is irrational.

Sol^n: First we will prove the following Lemma:

Lemma: $(\forall n \in \mathbb{Z})\ [2 \text{ divides } n^3 \Rightarrow 2 \text{ div } n]$

Proof: Let $(n \in \mathbb{Z})$ and $2 \mid n^3$

assume 2 does not divide n

$\Rightarrow n = 2q + 1$ (n is odd)

$\Rightarrow n^3 = 8q^3 + 12q^2 + 6q + 1$

$\Rightarrow n^3 = 2(4q^3 + 6q^2 + 3q) + 1$

$\Rightarrow n^3$ is odd and 2 cannot divide n^3

contradiction.

hence $(\forall n \in \mathbb{Z})\,[2 \mid n^3 \Rightarrow 2 \mid n]$

$\otimes$

now lets get back to the proof.

assume $(2)^{1/3}$ rational.

$\Rightarrow (2)^{1/3} = p/q$; $p, q \in \mathbb{Z}$; $q \neq 0$; $\gcd(p,q) = 1$

$\Rightarrow 2 = p^3/q^3$; $2q^3 = p^3$ — (1)

$\Rightarrow 2 \mid p^3$

$\Rightarrow$ by the Lemma $2 \mid p$

$\Rightarrow p = 2 \cdot m$

$\Rightarrow 2q^3 = (2m)^3$; $2q^3 = 8m^3$

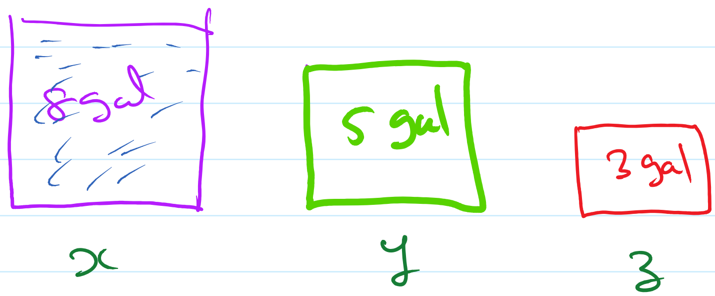
$\Rightarrow q^3 = 4m^3$

$\Rightarrow 2 \mid q^3 \Rightarrow 2 \mid q$ (Lemma)

Contradiction ($\gcd(p,q) = 1$)

$\therefore \ (2)^{1/3}$ is irrational. $\otimes$

#38



Let x : be the amount of water in the 8 gal jug.

y : " " " " " 5 gal "

z : " " " " " 3 gal "

Initial condition: $x = 8$; $y = 0$; $z = 0$

Step 1: $x = 5$; $y = 0$; $z = 3$

Step 2: $x = 5$; $y = 3$; $z = 0$

Step 3: $x = 2$; $y = 3$; $z = 3$

Step 4: $x = 2$; $y = 5$; $z = 1$

Step 4: $x = 2$; $y = 5$; $z = 0$ (throw the water out)

Step 5: $\underline{x = 2}$; $y = 2$; $z = 3$

step6: $\boxed{x=4}$; $y=0$; $z=1$