Note Title 1/15/2015

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$\boxed{\#6}$ Let $n, m \in \mathbb{Z}$ and n, m be odd

$\Rightarrow n = 2k+1$; $m = 2l+1$; where $k, l \in \mathbb{Z}$.

$\Rightarrow n * m = (2k+1)(2l+1)$

$\Rightarrow n * m = 4kl + 2l + 2k + 1$

$\Rightarrow n * m = 2(2kl + l + k) + 1$

hence by definition of "odd"

$n * m$ is odd

$\boxed{\#11}$ Let $r = \sqrt{2}$, $s = \sqrt{2}$ two irrational number

but $r * s = \sqrt{2} * \sqrt{2} = 2$ Rational

$\boxed{\text{False}}$

$\boxed{18}$ (a) show $(\forall n \in \mathbb{Z})\left[(3n+2) \text{ even} \Rightarrow n \text{ even}\right]$

Contraposition : $(\forall n \in \mathbb{Z})\left[n \text{ odd} \Rightarrow (3n+2) \text{ odd}\right]$

to Prove contraposition:

Let $(n \in \mathbb{Z})$ and n odd

$\Rightarrow n = 2k+1 \quad (k \in \mathbb{Z})$

$\Rightarrow 3n+2 = 3(2k+1) + 2$

$\Rightarrow 3n+2 = 6k+3+2 = 6k+4+1$

$\Rightarrow 3n+2 = 2(3k+2)+1$

here $(3n+2)$ odd

here $(\forall n \in \mathbb{Z})\left[(3n+2) \text{ even} \Rightarrow n \text{ even}\right]$

Note: an Easier proof:

$3n+2$ even $\Rightarrow$ $3n$ even $\Rightarrow$ since 3 is odd
n is even]

(b) Proof by contradiction:

Assume $(n \in \mathbb{Z})\left[(3n+2) \text{ even} \wedge n \text{ is odd}\right]$

$\Rightarrow (3n+2-n)$ is odd

$\Rightarrow (2n+2) = 2(n+1)$ is odd

contradiction.

[This is what we did:

$(\forall n \in \mathbb{Z})\left[\underbrace{(3n+2) \text{ even}}_{p} \Rightarrow \underbrace{n \text{ even}}_{q}\right]$]

we proved $\overset{T}{p \wedge \neg q \Rightarrow F} \equiv \sim(p \wedge \neg q) \vee F$

$\equiv \sim(p \wedge \neg q) \equiv \overset{T}{\sim p \vee q}$ True

clearly since we assumed p is true

$\sim p$ is false here $\sim p \vee q \equiv q$ true.

#38 we have to show that there is one such number. So we look for the smallest such number (Easy)

Try: $0 = 0^2 + 0^2 + 0^2$ (NO)

$1 = 1^2 + 0^2 + 0^2$ (no)

$2 = 1^2 + 1^2 + 0^2$ (no)

$3 = 1^2 + 1^2 + 1^2$ (no)

$4 = 2^2 + 0^2 + 0^2$ (no)

$5 = 1^2 + 2^2 + 0^2$ (no)

$6 = 1^2 + 2^2 + 1^2$ (no)

The smallest number is 7. But we have to argue that this works.

$7 = 6 + 1$; From the derivation of 6

the next possible number is

$$1^2 + 2^2 + 2^2 = 9$$

here 7 cannot be written as sum of three squares

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