COSC 545, Spring 2020: Problem Set $\#2$

Due: Thur., 2/20, at the beginning of class (hand in hard copy).
Covers: Lectures 7 to 11.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. In class we proved that there must exist languages that are not Turing recognizable by counting the number of Turing Machines and the number of possible languages. Explain briefly but clearly why the diagonalization argument we used to prove there are an uncountably infinite number of languages cannot be applied to prove that there are an uncountably infinite number of Turing Machines.

2. We say that a TM M has a *useless* state if there is some state q that is not the accept or reject state (i.e., $q \notin \{q_A, q_R\}$) such that M never enters q. That is, over all possible input strings, M never enters state q. Given an arbitrary TM M, describe how to modify M into a machine M' such that $L(M) = L(M')$, and M' does not have any useless states.

   (Hint: Modifying M into M' in this way *does not* mean you have to remove useless states from M. There are other ways to modify a TM's definition to ensure that it has no useless states.)

3. Consider the language $A = \{\langle M \rangle \mid M \text{ is a TM with a useless state}\}$. Prove A is undecidable using a reduction argument. (Hint: the answer I had in mind borrowed ideas from the answer to Problem 2).

4. Explain why we cannot apply Rice's Theorem to prove the language from Problem 3 is undecidable.

5. Use a mapping reduction to prove the following language is not decidable:
   $T = \{\langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ if and only if it accepts } w\}$.
   (Recall: w^R is the reverse of string w; e.g., if $w = cat$ then $w^R = tac$.)

6. Prove that $\overline{HALT_{TM}}$ is not Turing Recognizable.

7. Let L_{reg} be the set of regular languages. Let L_{LBA} be the set of languages that can be decided by LBAs. Prove that $L_{reg} \subset L_{LBA}$. That is, that LBA's are strictly more powerful than finite automata.