COSC 545, Spring 2020: Problem Set $\#1$

Due: Tue., 2/4, at the beginning of class (hand in hard copy).
Covers: Lectures 1 to 6 (all of Part 1).
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Describe clearly and succinctly how to modify our proof that regular languages are closed under the union operator to show that they are also closed under the following two related operators:

   (a) *Intersection:* $A \cap B = \{w \mid w \in A \wedge w \in B\}$

   (b) *Difference:* $A \setminus B = \{w \mid w \in A \wedge w \notin B\}$

2. Provide a formal proof that the regular languages are closed under the *complement* operator defined: $\bar{A} = \{w \mid w \notin A\}$. Use the same level of detail that we used for the union operator argument in class; i.e., given the formal definition of DFA $M = (Q, \Sigma, \delta, q_0, F)$, where $L(M) = A$, define the elements of DFA $M' = (Q', \Sigma, \delta', q_0', F')$, where $L(M') = \bar{A}$.

3. Using the technique presented in class to show that every regular expression is a regular language, construct an NFA that recognizes the same language as this regular expression: $(00)^*(11) \cup 01$.

4. For each of the following four languages, either prove it is non-regular (using the pumping lemma) or prove it regular (by drawing the state diagram of the finite automaton that accepts it):

   (a) $\{w \mid w \in \{0,1\}^* \setminus \{0, 1, 00\}\}$ (note: $\setminus$ is the *set minus* operator).

   (b) $\{w_1 w_2 w_1 \mid w_1, w_2 \in \{0,1\}^+\}$.

   (c) $\{a^i b^j \mid i, j > 0,$ i and j have different parities$\}$ (i.e., one is even and the other is odd).

   (d) $\{w\#y \mid w, y \in \{a,b\}^*, w \neq y\}$.
   (Note: The solution I have in mind for this final case is more involved than the solutions for the previous cases. You might find it useful to use what you proved in problem 1 about intersection and complement.)

5. Consider the following language of binary palindromes: $\{w \mid w \in \{0,1\}^*, w = w^{\mathcal{R}}\}$. (Recall that $w^{\mathcal{R}}$ is the string w backwards; e.g., if $w = 111000$ then $w^{\mathcal{R}} = 000111$.)

   (a) Give a CFG that generates this language.

   (b) Draw the state diagram for a PDA that accepts this language.