COSC 545, Spring 2014: Problem Set #4

Due: Thur., 4/10, at the beginning of class (hand in hard copy).
Covers: Lectures 19 to 22.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Let ANA $= \{\langle w_1, w_2\rangle \mid w_1$ and w_2 are anagrams$\}$. Assume I am able to prove TQBF $\leq_p$ ANA. Show that this implies PSPACE = P.

2. Let $EQ_{NFA} = \{\langle N_1, N_2 \mid N_1$ and N_2 are NFAs and $L(N_1) = L(N_2)\}$. Prove that $EQ_{NFA} \in PSPACE$. (In your answer, you may find it useful to leverage Savitch's Theorem.)

3. In class and in Theorem 8.14 of the Sipser textbook we proved that GG (Generalized Geography) is $PSPACE$-complete. This proof required us to construct a graph structure of the form shown in Figure 8.16. Consider the following update to this graph structure:

   > In the original proof, we include a column of diamonds gadgets, one for each variable x_i. (This structure is highlighted in Figure 8.15.) In more detail, each x_i has a 4-node diamond, consisting of a *top* node with directed edges to a *left* and *right* node, each with a directed edge to the *bottom* node.
   >
   > In our new version of the structure, we replace each diamond with a hexagon. In more detail, we point the outgoing edge from *left* to a new node, *left2*, then point an outgoing edge from *left2* to the *bottom* node. We add a *right2* node in a similar fashion. The gadgets are otherwise connected as in the original proof, that is: the *bottom* of one gadget connects to the *top* below, and the edges incoming from the right side can stay the same (each goes to a single *left* or *right* node).

   Describe where the proof of Theorem 8.14 (GG is $PSPACE$-complete) breaks when we switch to this modified version of the graph structure.

4. Prove that $A_{DFA} = \{\langle D, w\rangle \mid D$ is a DFA that accepts string $w\}$ is in L.