COSC 545, Spring 2014: Problem Set $\#1$

Due: Tue., 1/23, at the beginning of class (hand in hard copy).
Covers: Lectures 1 to 4.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Provide formal arguments that regular languages are closed under *intersection* ($A \cap B = \{w \mid w \in A \wedge w \in B\}$) and *complement* ($\bar{A} = \{w \mid w \notin A\}$). Your arguments should use the same level of formal detail as the proof of closure under union presented in the textbook.

2. For each of the following four languages, either prove it is non-regular (using the pumping lemma) or prove it regular (by drawing the state diagram of the finite automaton that accepts it):

   (a) $\{w \mid w \in \{0,1\}^* \setminus \{0,1,00\}\}$ (note: $\setminus$ is the *set minus* operator).

   (b) $\{w_1 w_2 w_1 \mid w_1, w_2 \in \{0,1\}^*\}$.

   (c) $\{a^i b^j \mid i, j > 0,\, i$ and j have different parities$\}$ (i.e., one is even and the other is odd).

   (d) $\{w\#y \mid w, y \in \{a,b\}^*, w \neq y\}$.
   (Note: The solution I have in mind for this final case is more involved than the solutions for the previous cases. You might find it useful to use what you proved in problem 1 about intersection and complement.)

3. Give a CFG generating the language of binary strings that are palindromes (i.e., read the same forward and backward).

4. Let a *neat PDA* be a variant of a PDA that only accepts a string if the machine ends in an accept state *with* an empty stack. Draw the state diagram for a *one state* neat PDA (your state diagram should only have one state) that accepts the language of strings from $\Sigma = \{a,b\}$ with an equal number of a and b's.