COSC 545, Spring 2012: Problem Set #6

Due: Wed., 4/25, at the beginning of class (hand in hard copy).
Covers: Lectures 20 to 24.
(Note: *we might not get to the material covered in lecture 24 until the week the problem set is due. You might plan to read the relevant chapter in advance on your own.*)
Collaboration: You may collaborate with classmates. Every student must write up his or her own answers and list collaborators. No sources outside of the assigned textbook may be consulted.

Note: The first 3 problems are easy to medium difficulty, while the last problem is (or, should be) harder. Do not wait until the last minute to start it.

Problems

1. ***NL-Completeness:*** Show that A_{NFA} is NL-complete.

2. ***Time Hierarchy Theorem:*** Problem 9.12 from Sipser.

3. ***Relativization:*** Prove that there exists an oracle A such that $\text{NP}^A \neq \text{coNP}^A$.
   (Note: *coNP^A contains every language L such that the complement of L is solvable in non-deterministic polynomial time using oracle A.*)

4. ***Approximation:*** The Traveling Salesman Problem (TSP) is an optimization problem that takes as input a weighted complete undirected graph, and outputs the minimum cost tour of the graph (where a tour is a path that visits every node exactly once, and the cost of a tour is the sum of its edge weights).[1] Karp proved TSP to be NP-Complete. Here we explore the possibility of approximation.

   Prove that for any constant approximation factor $\alpha \geq 1$, if P $\neq$ NP, then there does *not* exist a polynomial time α-approximation algorithm for TSP.
   (Note: *In your answer, the* **only** *problems you can assume to be NP-complete are those covered in Sections 7.4 and 7.5.*)
   (Hint: *You might find it hard to connect the version of TSP described in this problem directly to an NP-complete problem from Sipser. One useful strategy is to first transform a solution of the TSP variant described above to a variant more easily connected to the NP-complete problems of 7.4 and 7.5. Any such transformation, however, must be spelled out in detail.*)

[1] There are many different variants of TSP, of which this is only one. You must, however, use this exact variant in your solution. If you find another variant to be easier to work with, you must explicitly reduce the easier version to the variant described here.