COSC 545, Spring 2012: Problem Set $\#4$

Due: Wed., 3/28, at the beginning of class (hand in hard copy).
Covers: Lectures 14 to 17.
Collaboration: You may collaborate with classmates. Every student must write up his or her own answers and list collaborators. No sources outside of the assigned textbook may be consulted.

Note: To keep things fresh, I am trying something different with this problem set. Instead of offering $6-7$ problems of easy to medium difficulty, I am instead offering 3 problems of easy to medium difficulty (problems $1-3$) and one problem that I believe to be of *hard* difficulty (problem 4).

Problems

1. ***The P Complexity Class:*** In the following problem, let $mix(S)$, for set S of symbols, be the set consisting of every string made up of symbols from S, such that no symbol appears more than once in the string. Fix some CFG $G = (V, \Sigma, R, S)$. Prove that the language $A_G = \{S \mid S \subseteq \Sigma, \forall s \in mix(S) : s \in L(G)\}$ is in P.

2. ***NP-Completeness:*** Problem 7.17 from Sipser.

3. ***More NP-Completeness:*** Problem 7.20 from Sipser.
(*Note:* The definitions of languages PATH and UHAMPATH can be found in Chapter 7 of Sipser; also, a *simple path* in a graph is a path in which no node is repeated.)

4. ***Tricky NP-Completeness:*** We say an assignment to a 3cnf-formula is *balanced* if the assignment satisfies the formula and at least one literal in each clause evaluates to 0. Let $BSAT$ be the collection of 3cnf-formulas that have a balanced assignment. Prove that $BSAT$ is NP-complete.