# COSC 545, Spring 2012: Problem Set $\#2$

**Due:** Wed., 2/15, at the beginning of class (hand in hard copy).
**Covers:** Lectures 5 to 8.
**Collaboration:** You may collaborate with classmates. Every student must write up his or her own answers and list collaborators. No sources outside of the assigned textbook may be consulted.

**A Note on TM Description Formality:** As described in class and in the Sipser textbook, there are different levels of detail with which you can describe a Turing machine. The most possible detail is defining every object of the formal mathematical description. The least amount of detail is a high-level description of an algorithm.

*For this assignment*, with the exception of problem 1 (which asks for a state machine), the appropriate amount of detail to use when describing a TM is the same level used in Example 3.7 from Chapter 3.1 of Sipser (e.g., describe, in words, the behavior of the TM).

## Problems

1. ***Practicing Formal TM Definition:*** Provide the state machine diagram for a deterministic single-tape TM that decides the language $\{w \in \{(,)\}^* \mid w \text{ describes properly nested parentheses}\}$. In this problem, assume the input alphabet $\Sigma = \{(,)\}$. You can define the tape alphabet $\Gamma$ however you want. In addition to your state diagram, also provide a brief text description of your TM's behavior, to aid the grader.

2. ***Equivalence of TM Variants:*** 3.11, from Sipser. (You may use any result proved in Chapter 3.2 of Sipser.)

3. ***More Equivalence of TM Variants:*** Exercise 3.3 asks that you modify the proof of Theorem 3.16 (each non-deterministic TM has an equivalent deterministic TM) to show that a language is decidable iff a non-deterministic TM decides it. The solution to this exercise is given in the *Selected Solutions* for the chapter. This sample solution, however, is missing details. This problem asks you to fill in some of these missing details:

   (a) The description of Exercise 3.3 includes a tree theorem that you are allowed to assume. This theorem requires two assumptions about a tree before it can be applied. Argue why both of these assumptions are true in the context where the theorem is applied in the sample solution for Exercise 3.3.

   (b) Modify the description the TM $D$ from the proof of Theorem 3.16 to implement the new Stage 5 described in the sample solution.

4. ***Decidability:*** 4.12, from Sipser. (You may use any result proved in Chapter 4 of Sipser, as well as any result about regular languages proved in class, earlier problem sets, or the textbook.)

5. ***Connections Between Decidability and Recognizability:*** 4.17, from Sipser.

6. ***Mapping Reducibility:*** This problem has two parts. In the following, you can assume the input alphabet $\Sigma = \{0, 1\}$.

(a) 5.24, from Sipser.

(b) Show that for the language $J$ from part (a): $J \leq_m \bar{J}$.

7. ***Turing Reducibility:*** We say a language $L'$ is a *superior twin* for language $L$ if: $L \leq_T L'$ and $L' \not\leq_T L$. Prove that every language has a superior twin.