COSC 030, Fall 2016: Problem Set #2

Assigned: Tuesday, 9/13.
Due: Tuesday, 9/20, at the beginning of class (hand in hard copy).
Lectures Covered: End of Week 2, Week 3.
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. A *prime* number is whole number greater than 1 that is only divisible by 1 and itself (e.g., 2,3,5,7,11,13,17...). A useful property of prime numbers is that they are always odd.

   - Prove this property using a proof by *contraposition*.
   - Prove this property using a proof by *contradiction*.

2. Let $A = \{a, b, c\}$, $B = \{dog, cat, house, a\}$, and $C = \{a, cat\}$. Specify whether each of the following statements is *true* or *false*.

   (a) $C \subset B$
   (b) $A \subseteq B$
   (c) $C \subseteq C$
   (d) $(\mathbb{R} - A) \cup C$ is finite.
   (e) $|A \cup C| = |B|$

3. Describe using set builder notation a set D such that: (a) D is infinite; and (b) $|B - D| = 1$ (where B is defined in the preceding problem).

4. Define a function f that satisfies the following three properties: (1) its domain and range both contain 3 elements; (2) it is one-to-one; and (3) it is *not* onto.

5. Let $\mathbb{D}_{\mathbb{E}}$ be the set of all words in the standard English dictionary. Let $h : \mathbb{D}_{\mathbb{E}} \to \{a, b, c, ..., y, z\}$ be the function that maps each word $w \in \mathbb{D}_{\mathbb{E}}$ to the first letter in w (e.g., $h(cat) = c$). Let $\{a_i\}$, be the sequence defined for $1 \leq i \leq 3$ where a_1 = math, a_2 = methods, a_3 = problem.

   What is the value of the following summation:

   $$\sum_{i=2}^{3} |\{h(a_i)\} \cap \{m\}|$$