COSC 030, Fall 2015: Problem Set $\#9$

Assigned: Tuesday, 11/17.
Due: Thursday, 12/3. **Lectures Covered:** Week 12 to 14 (Chapters 10.1 to 10.6).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Draw each of the following graphs:

   - K_4
   - $K_{9,2}$
   - W_3
   - Q_2

2. Let S be the set containing every undirected pseudograph with 3 edges and the vertex set $V = \{a, b, c, d\}$. What is the size of S?

3. Let S be the set containing every simple undirected graph defined over the vertex set $V = \{a, b, c, d, e\}$. Assume I select a graph from S with uniform randomness (that is, every graph is selected with probability $1/|S|$). What is the probability that I select a graph in which node a has no neighbors?

4. Use the pigeonhole principle to prove that no simple undirected graph with n vertices has a matching consisting of *more* than $n/2$ edges.

5. Prove that every bipartite graph with $n > 1$ vertices has a vertex cut of size $\leq n/2$.

   (There is no need to use induction.)

6. Draw a graph G with 5 vertices such that $\chi(G) = 4$.

7. In class we proved that in an m-ary tree of height h and ℓ leaves, $h \geq \lceil \log_m(\ell) \rceil$. Prove that for any $m \geq 2$ and $\ell \geq 2$ there exists a tree with $h \geq \ell$.

   (There is no need to use induction. It is sufficient to describe how to construct a graph with the stated property for any provided m and ℓ.)