Linear Models for Classification: Features & Weights

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(some slides borrowed from Chris Dyer)
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Outline

- **Words, probabilities → Features, weights**
- **Geometric view: decision boundary**

this lecture

- Perceptron

next lecture

- Generative vs. Discriminative
- More discriminative models: Logistic regression/MaxEnt; SVM
- Loss functions, optimization
- Regularization; sparsity

Word Sense Disambiguation (WSD)

- Given a word **in context**, predict which sense is being used.
  - Evaluated on corpora such as **SemCor**, which is fully annotated for WordNet synsets.

- For example: consider joint POS & WSD classification for 'interest', with 3 senses:
  - N:financial (*I repaid the loan with interest*)
  - N:nonfinancial (*I read the news with interest*)
  - V:nonfinancial (*Can I interest you in a dessert?*)

Beyond BoW

- Neighboring words are relevant to this decision.
- More generally, we can define **features** of the input that may help identify the correct class.
  - Individual words
  - Bigrams (pairs of consecutive words: *Wall Street*)
  - Capitalization (*interest* vs. *Interest* vs. *INTEREST*)
  - Metadata: document genre, author, ...
- These can be used in naïve Bayes: "bag of features"
  - With overlapping features, independence assumption is *even more naïve*: $\mathrm{p}(y \mid \mathbf{x}) \propto \mathrm{p}(y) \cdots \mathrm{p}(\text{Wall} \mid y)\, \mathrm{p}(\text{Street} \mid y)\, \mathrm{p}(\text{Wall Street} \mid y)$

Choosing Features

- Supervision means that we don't have to pre-specify the precise relationship between each feature and the classification outcomes.
- But domain expertise helps in choosing which kinds of features to include in the model. (words, subword units, metadata, ...)
  - And sometimes, highly task-specific features are helpful.
- The decision about what features to include in a model is called **feature engineering**.
  - (There are some algorithmic techniques, such as *feature selection*, that can assist in this process.)
  - More features = more flexibility, but also more expensive to train, more opportunity for overfitting.

Feature Extraction

$\mathbf{x}$ = Wall Street vets raise concerns about interest rates , politics

	$\phi(\mathbf{x})$
bias	1
capitalized?	0
#wordsBefore	6
#wordsAfter	3
relativeOffset	0.66
leftWord=about	1
leftWord=best	0
rightWord=rates	1
rightWord=in	0
Wall	1
Street	1
vets	1
best	0
in	0
Wall Street	1
Street vets	1
vets raise	1
...	

bias feature (≈class prior): value of 1 for every $\mathbf{x}$ so the learned weight will reflect prevalence of the class

- Turns the input into a table of features with real values (often binary: 0 or 1).
- In practice: define feature templates like "leftWord=•" from which specific features are instantiated

Feature Extraction

$\mathbf{x}$ = Wall Street vets raise concerns about interest rates , politics

spelling feature

	$\phi(\mathbf{x})$
bias	1
capitalized?	0
#wordsBefore	6
#wordsAfter	3
relativeOffset	0.66
leftWord=about	1
leftWord=best	0
rightWord=rates	1
rightWord=in	0
Wall	1
Street	1
vets	1
best	0
in	0
Wall Street	1
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...	

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...	

token positional features

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relativeOffset	0.66
leftWord=about	1
leftWord=best	0
rightWord=rates	1
rightWord=in	0
Wall	1
Street	1
vets	1
best	0
in	0
Wall Street	1
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...	

immediately neighboring words

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relativeOffset	0.66
leftWord=about	1
leftWord=best	0
rightWord=rates	1
rightWord=in	0
Wall	1
Street	1
vets	1
best	0
in	0
Wall Street	1
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…	

unigrams

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Feature Extraction

	$\phi(\mathbf{x})$
bias	1
capitalized?	0
#wordsBefore	6
#wordsAfter	3
relativeOffset	0.66
leftWord=about	1
leftWord=best	0
rightWord=rates	1
rightWord=in	0
Wall	1
Street	1
vets	1
best	0
in	0
Wall Street	1
Street vets	1
vets raise	1
. . .	

bigrams

$\mathbf{x}$ = Wall Street vets raise concerns about interest rates , politics

- Turns the input into a table of features with real values (often binary: 0 or 1).
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Feature Extraction

$\mathbf{x}$ = Wall Street vets raise concerns about interest rates , politics

$\mathbf{x}'$ = Pet 's best interest in mind , but vets must follow law

	$\phi(\mathbf{x})$	$\phi(\mathbf{x}')$
bias	1	1
capitalized?	0	0
#wordsBefore	6	3
#wordsAfter	3	8
relativeOffset	0.66	0.27
leftWord=about	1	0
leftWord=best	0	1
rightWord=rates	1	0
rightWord=in	0	1
Wall	1	0
Street	1	0
vets	1	1
best	0	1
in	0	1
Wall Street	1	0
Street vets	1	0
vets raise	1	0

...

- Turns the input into a table of features with real values (often binary: 0 or 1).
- In practice: define feature templates like “leftWord=•” from which specific features are instantiated

Linear Model

- For each input $\mathbf{x}$ (e.g., a document or word token), let $\boldsymbol{\phi}(\mathbf{x})$ be a function that extracts a vector of its features.
  - Features may be binary (e.g., `capitalized?`) or real-valued (e.g., `#word=debt`).

- Each feature receives a real-valued **weight** parameter w. Each candidate label y' is scored for the token by summing the weights for the active features:

$$\mathbf{w}_{y'}{}^{\top}\boldsymbol{\phi}(\mathbf{x})$$
$$= \Sigma_j \, w_{y',j} \cdot \phi_j(\mathbf{x})$$

- For binary classification, equivalent to: $\mathrm{sign}(\mathbf{w}^{\top}\boldsymbol{\phi}(\mathbf{x}))$ — $+1$ or -1

	$\phi(\mathbf{x})$	w	$\phi(\mathbf{x}')$
bias	1	−3.00	1
capitalized?	0	.22	0
#wordsBefore	6	−.01	3
#wordsAfter	3	.01	8
relativeOffset	0.6	1.00	0.2
leftWord=about	1	.00	0
leftWord=best	0	−2.00	1
rightWord=rates	1	5.00	0
rightWord=in	0	−1.00	1
Wall	1	1.00	0
Street	1	−1.00	0
vets	1	−.05	1
best	0	−1.00	1
in	0	−.01	1
Wall Street	1	4.00	0
Street vets	1	.00	0
vets raise	1	.00	0

. . .

$\mathbf{x}$ = Wall Street vets raise concerns about interest rates , politics

$\mathbf{x}'$ = Pet 's best interest in mind , but vets must follow law

- Weights are learned from data
- For the moment, assume binary classification: financial or nonfinancial
  - More positive weights more indicative of financial.
  - $\mathbf{w}^{\mathsf{T}}\phi(\mathbf{x}) = 6.59$, $\mathbf{w}^{\mathsf{T}}\phi(\mathbf{x}') = -6.74$

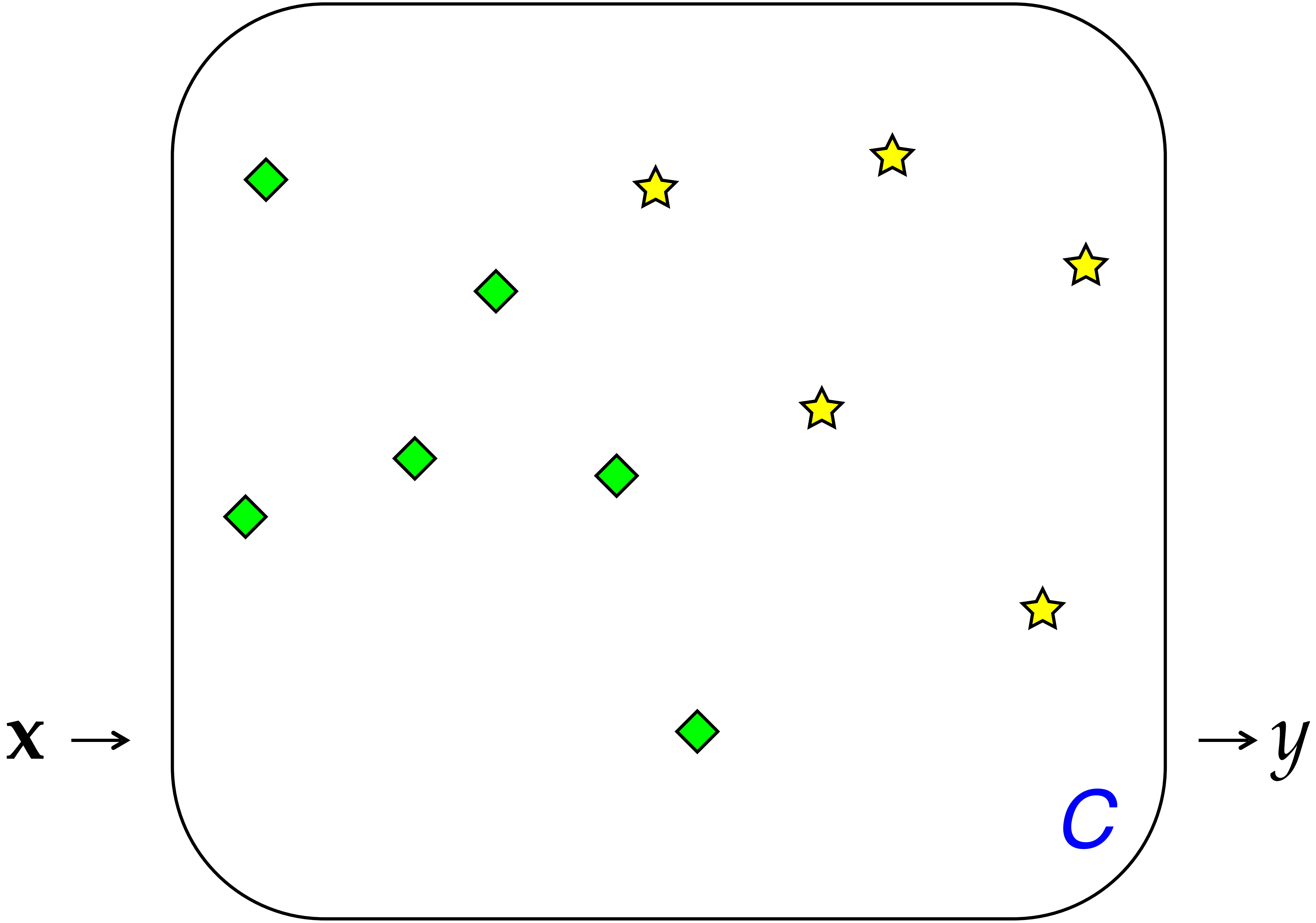
More than 2 classes

- Simply keep a separate weight vector for each class: $\mathbf{w}_y$
- The class whose weight vector gives the highest score wins!

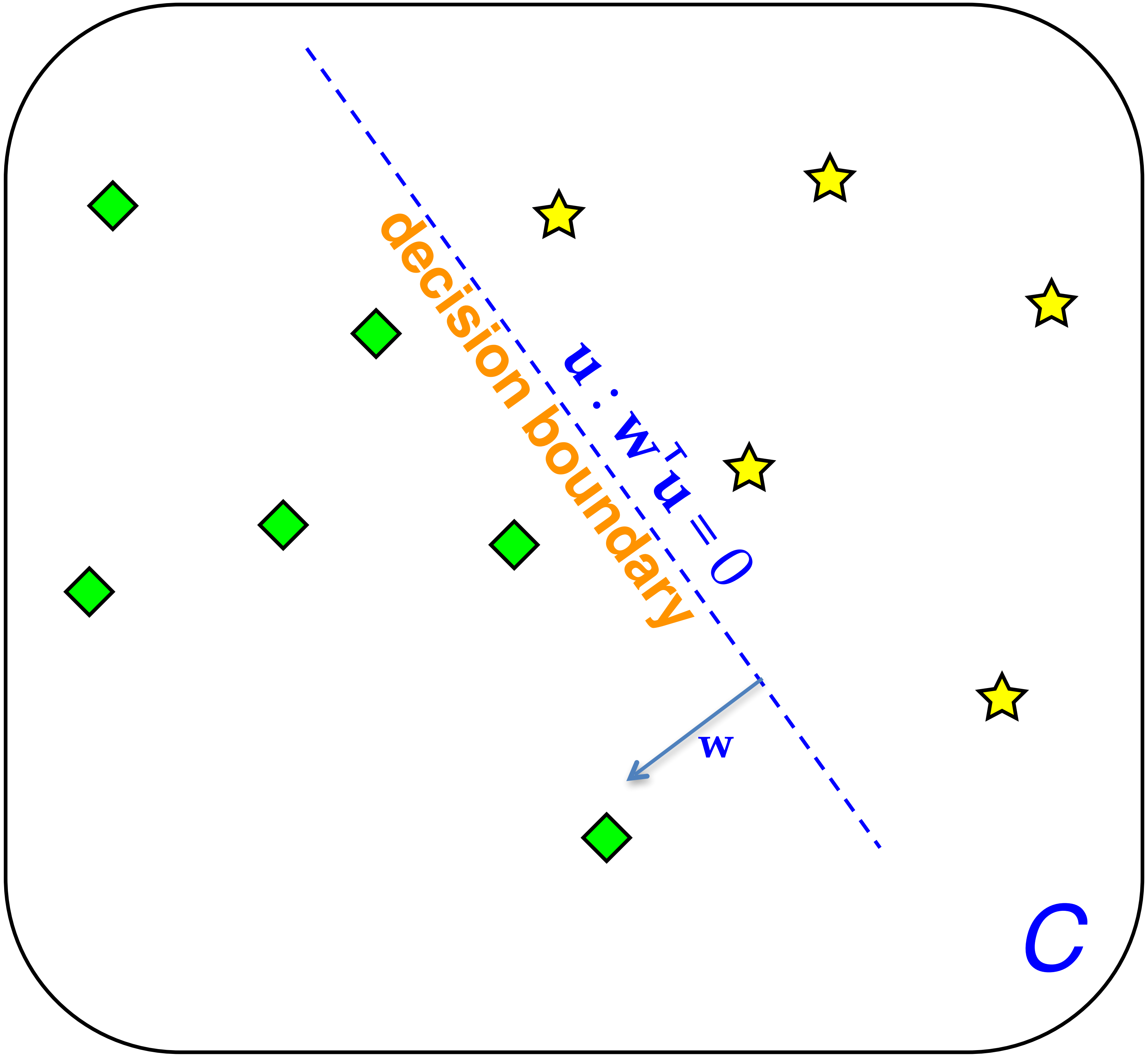
Learning the weights

- Weights depend on the choice of model and learning algorithm.
- Naïve Bayes fits into this framework, under the following estimation procedure for $\mathbf{w}$:
  - $w_{\text{bias}} = \log \mathrm{p}(y)$
  - $\forall$ features f: $w_f = \log \mathrm{p}(f \mid y)$
  - $\Sigma_j\, w_j \cdot \phi_j(\mathbf{x}) = w_{\text{bias}} + \Sigma_{f \in \phi(\mathbf{x})}\, w_f$
    $= \log \mathrm{p}(y) + \Sigma_{f \in \phi(\mathbf{x})} \log \mathrm{p}(f \mid y)$
    $= \log (\mathrm{p}(y) \cdot \Pi_{f \in \phi(\mathbf{x})}\, \mathrm{p}(f \mid y))$
- However, the naïve independence assumption—that all features are conditionally independent given the class—can be harmful.
  - Could the weights shown on the previous slide be naïve Bayes estimates?
    - No, because some are positive (thus not log-probabilities). Other kinds of learning procedures can give arbitrary real-valued weights.
    - If using log probabilities as weights, then the classification threshold should be equivalent to probability of .5, i.e. **log .5**.

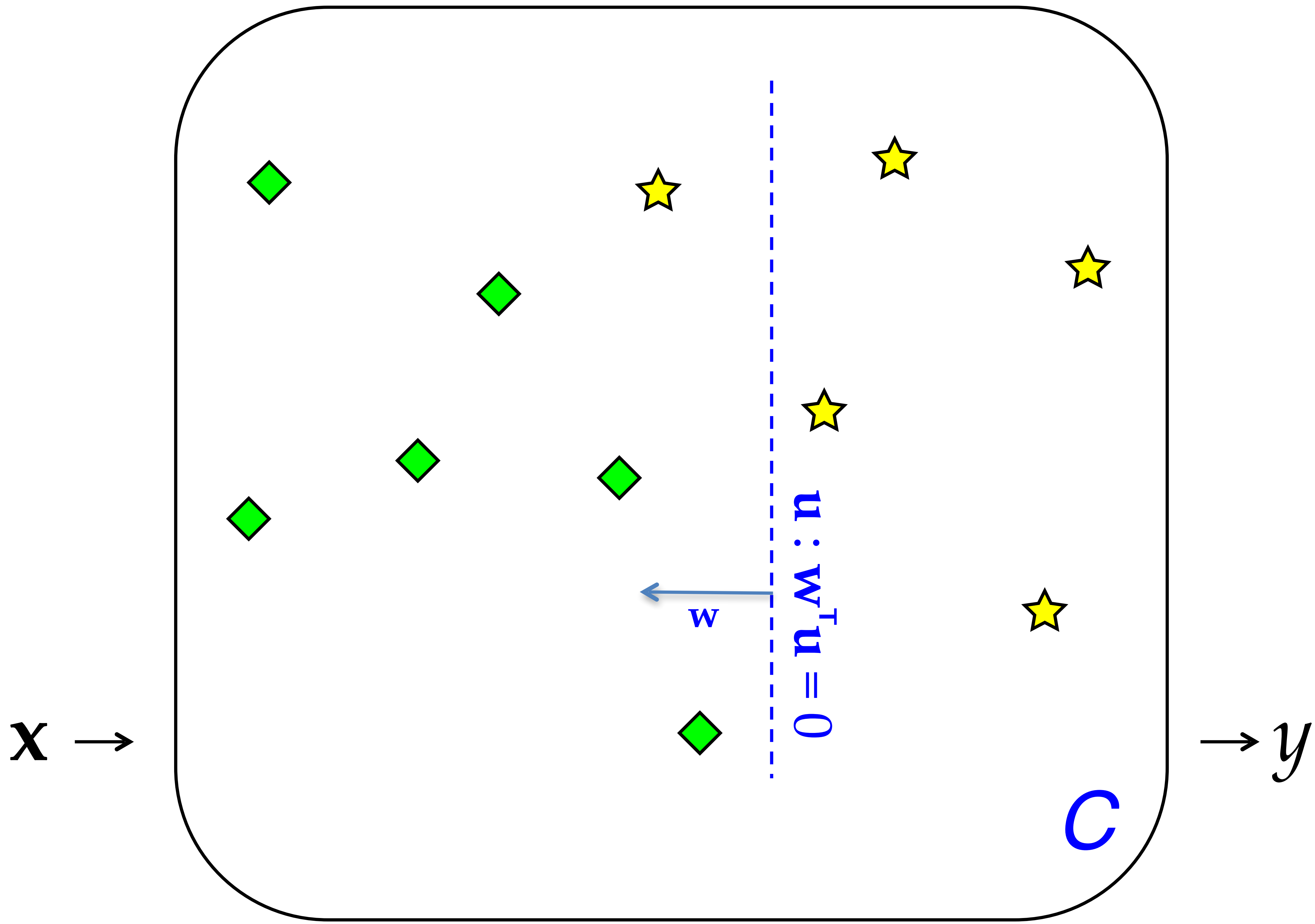
Linear Classifiers: Geometric View



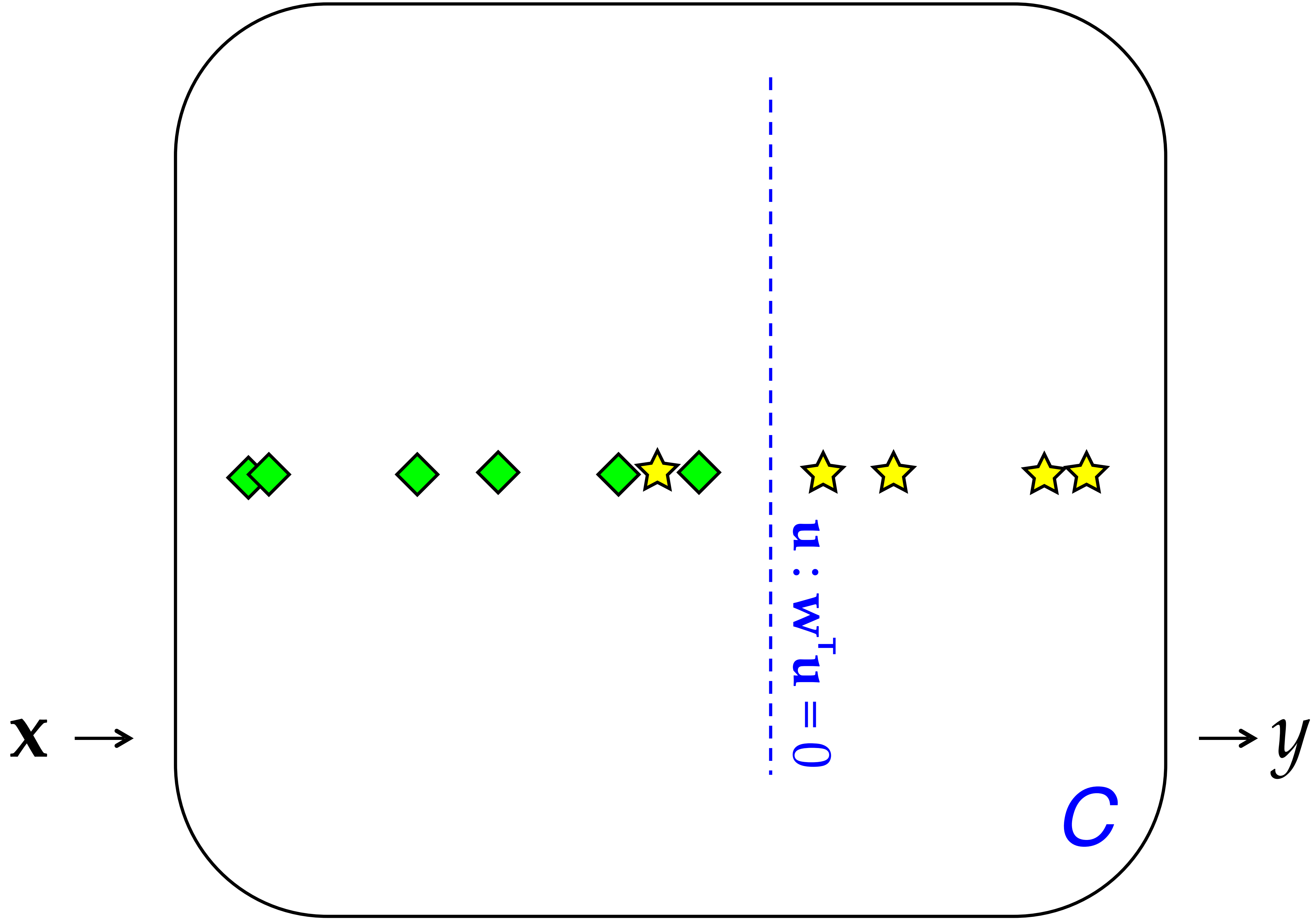
Linear Classifiers: Geometric View

$\mathbf{x} \rightarrow$  $\rightarrow y$

Linear Classifiers: Geometric View



Linear Classifiers: Geometric View



Linear Classifiers (> 2 Classes)

return

$\arg\max_y \mathbf{w}_y^\top \mathbf{\Phi}(\mathbf{x})$

$\mathbf{x} \rightarrow$ C $\rightarrow y$

The term "feature"

- The term "feature" is overloaded in NLP/ML. Here are three different concepts:
  - **Linguistic feature:** in some formalisms, a symbolic property that applies to a unit to categorize it, e.g. [−voice] for a sound in phonology or [+past] for a verb in morphology.
  - **Percept** (or **input feature**)**:** captures some aspect of an input $\mathbf{x}$; binary- or real-valued. *[The term "percept" is nonstandard but I think it is useful!]*

    ends in *-ing*
  - **Parameter** (or **model feature**): an association between some percept and an output class (or structure) y for which a real-valued weight or score is learned.

    ends in *-ing* $\wedge$ y=VERB