Classification: Naïve Bayes

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(slides adapted from Chris Dyer, Noah Smith, Sharon Goldwater, et al.)

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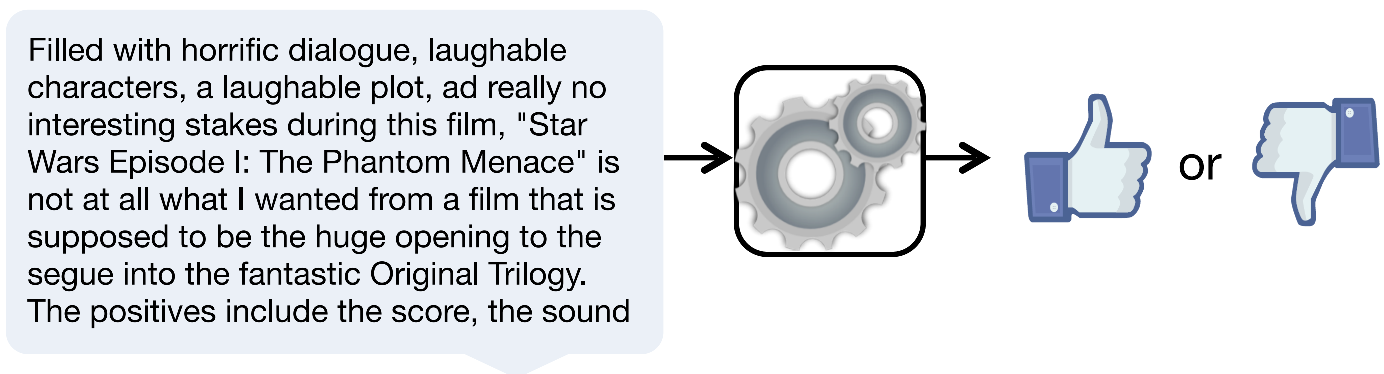
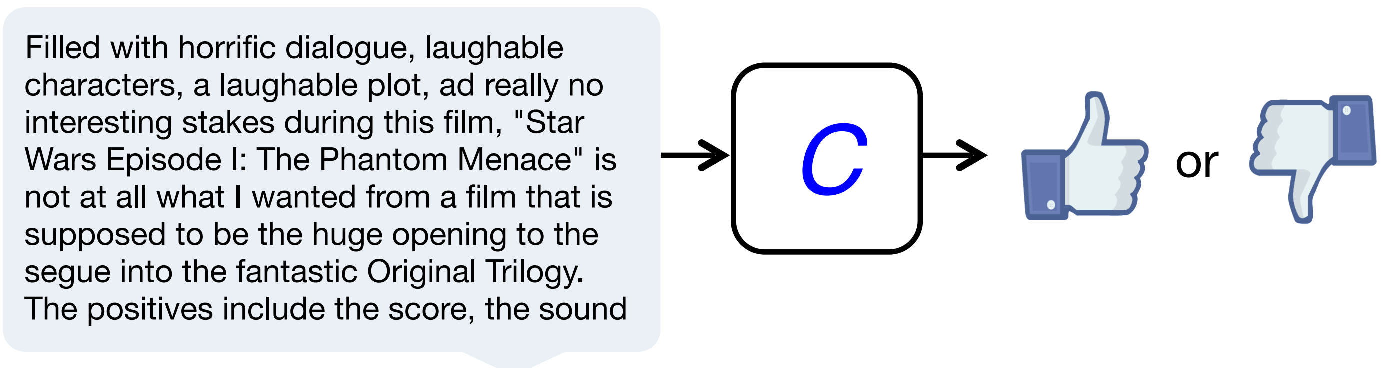
Sentiment Analysis

- Recall the task:



Sentiment Analysis

- Recall the task:



- This is a **classification** task: we have open-ended text as *input* and a fixed set of discrete classes as *output*.

- By convention, the input/observed information is denoted x, and the output/predicted information is y.

A Rule-based Classifier

```
good = {'yay', 'cool', …}
bad = {'ugh', ':(', …}

score = 0
for w in x:
  if w in good:
    score += 1
  elif w in bad:
    score -= 1
return int(score>0)
```

$x \rightarrow$ C $\rightarrow y$

Supervised Classification

- We can probably do better with data
  - Our intuitions about word sentiment aren't perfect
- **Supervised** = generalizations are **learned** from **labeled** data
  - So, we need a **training** corpus of reviews with gold (correct) sentiment labels
  - And a learning algorithm
- This course: **inductive** learning algorithms—collect statistics from training corpus, but the resulting classifier does not rely on the training corpus itself

A ~~Rule-based~~ Supervised Classifier

$x \rightarrow$

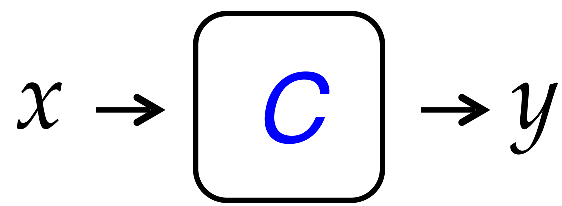
```
good = {...from training data...}
bad = {...from training data...}

score = 0
for w in x:
  if w in good:
    score += 1
  elif w in bad:
    score -= 1
return int(score>0)
```

C

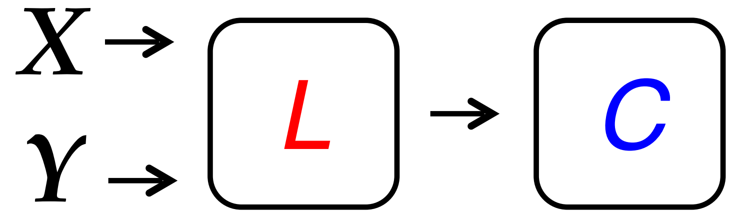
$\rightarrow y$

Notation

- Training examples: $\boldsymbol{X} = (x_1, x_2, \ldots, x_N)$
- Their categories: $\boldsymbol{Y} = (y_1, y_2, \ldots, y_N)$
- A **classifier** *C* seeks to map x_i to y_i:



- A **learner** *L* infers *C* from $(\boldsymbol{X}, \boldsymbol{Y})$:



Counting as Learning

$X \rightarrow$

$Y \rightarrow$

```
from collections import Counter
scores = Counter()
for x,y in zip(X,Y):
   for w in x:
      if y==THUMBS_UP:
         scores[w] += 1
      elif y==THUMBS_DOWN:
         scores[w] -= 1
good, bad = set(), set()
for w,score in scores.items():
   if score>0: good.add(w)
   else: bad.add(w)
return good, bad
```

L

$\rightarrow C$

Limitations

- Our classifier doesn't know that:
  - Some words are more strongly indicative of sentiment than others
  - The data may skew positive or negative (e.g., more or longer positive reviews than negative)
  - Infrequent words may occur only in the positive examples or only in the negative examples by accident
- Instead of raw counts, we can use a **probabilistic model**

Review Questions: Conditional Probability

1. If p is a probability mass function, which is true by the definition of conditional probability:
   $p(x \mid y, z) =$

   a. $p(x)/p(y,z)$

   b. $p(y)p(z)/p(x,y,z)$

   c. $p(x,y,z)/p(y,z)$

   d. $p(x)p(x|y)p(x|z)$

Review Questions: Conditional Probability

2. Which is/are guaranteed to be true?

   a. $\forall y\ \forall z,\ \Sigma_x\ p(x \mid y, z) = 1$

   b. $\forall x,\ \Sigma_y\ \Sigma_z\ p(x \mid y, z) = 1$

   c. $\Sigma_x\ p(x) = 1$

   d. $\forall y\ \forall z,\ \Sigma_x\ p(x)p(y \mid x)p(z \mid x, y) = 1$

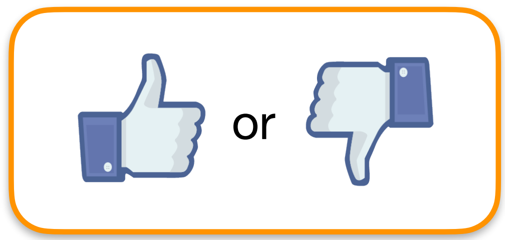
Probabilistic Classifiers

return

$\arg\max_{y'} p(y' \mid x)$

$x \rightarrow$ C $\rightarrow y$

Probabilistic Classifiers

return 

$\arg\max_{y'} p(y' \mid x)$

$x \rightarrow$ C $\rightarrow y$

Probabilistic Classifiers

$x \rightarrow$

return

$\arg\max_{y'} \mathrm{p}(y' \mid x)$

Filled with horrific dialogue, laughable characters, a laughable plot, ad really no interesting stakes during this film, "Star Wars Episode I: The Phantom Menace" is not at all what I wanted from a film that is supposed to be the huge opening to the segue into the fantastic Original Trilogy. The positives include the score, the sound

C

$\rightarrow y$

Probabilistic Classifiers

$x \rightarrow$

return

$\arg\max_{y'} \text{p}(y' \mid x)$

$= \text{p}(y' \mid$ Filled, with, horrific, …$)$

$\rightarrow y$

How can we compute this?

C

We can't compute the usual MLE unless this exact document appeared in the training data!

A probabilistic model that generalizes

- Instead of estimating $p(y' \mid \text{Filled, with, horrific, ...})$ directly, we make two **modeling assumptions**:

  1. The **Bag of Words (BoW) assumption:** Assume the order of the words in the document is irrelevant to the task. I.e., stipulate that $p(y' \mid \text{Filled, with, horrific}) = p(y' \mid \text{Filled, horrific, with})$

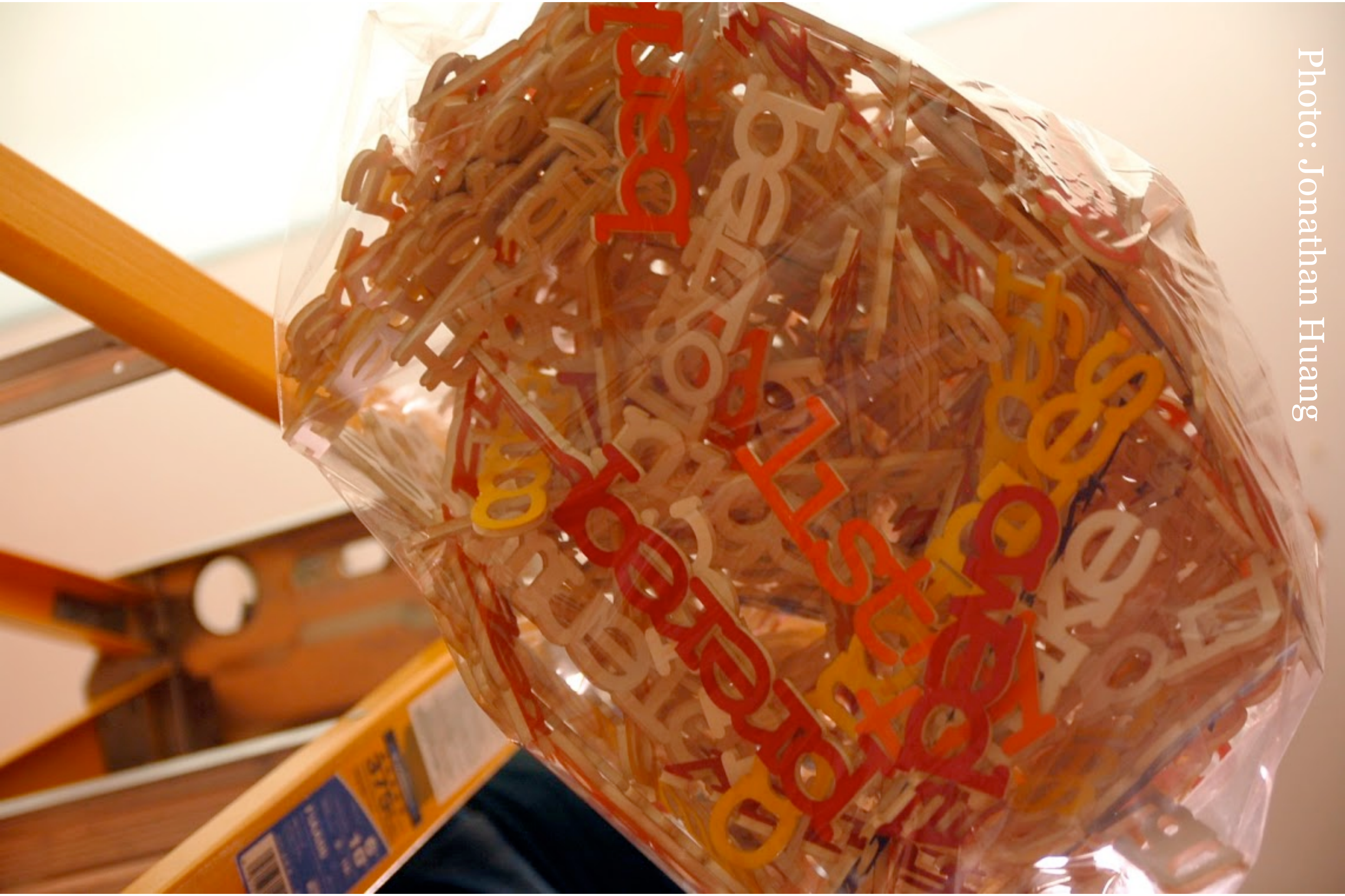


Photo: Jonathan Huang

Art installation in CMU’s Machine Learning Department

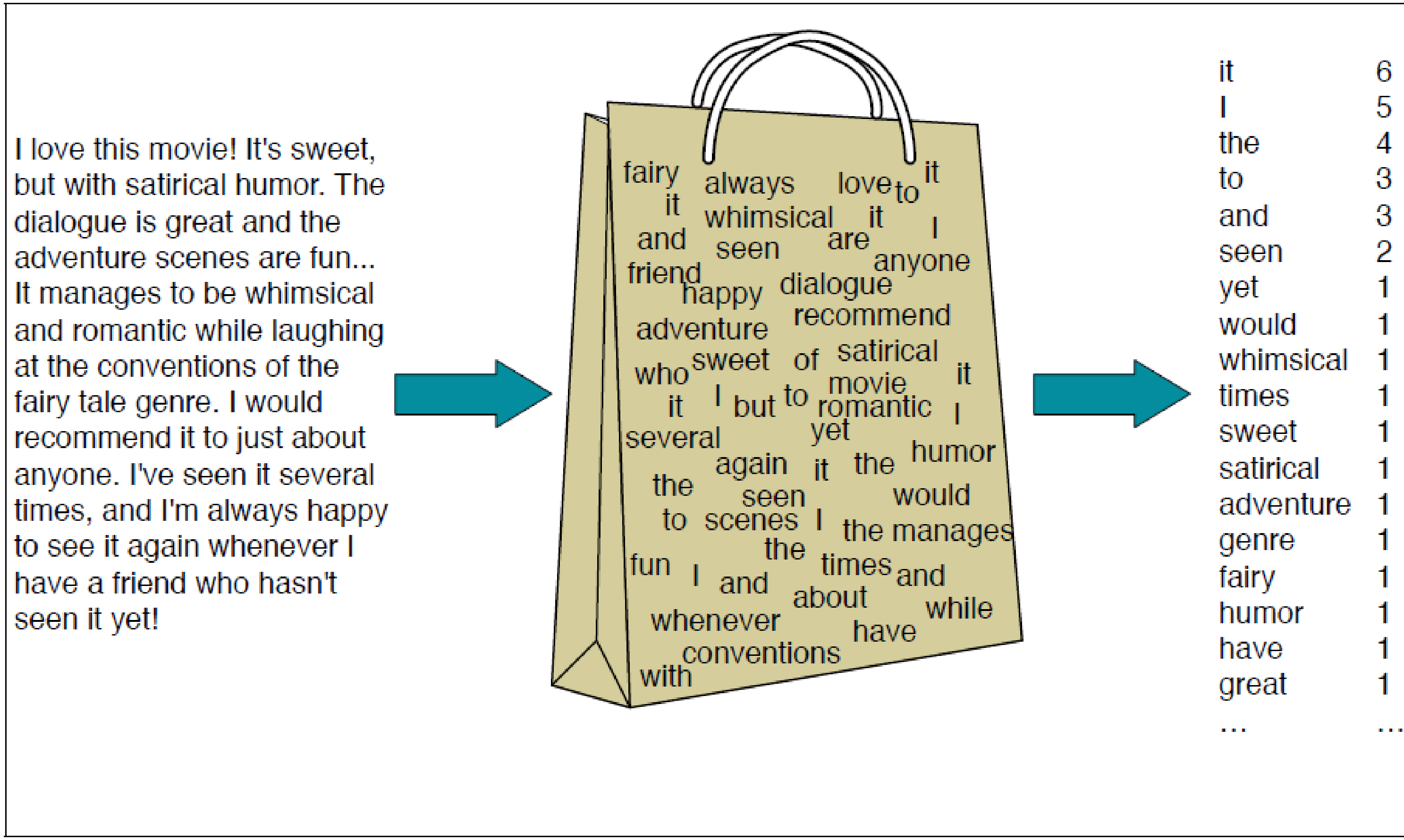


Figure 7.1 Intuition of the multinomial naive Bayes classifier applied to a movie review. The position of the words is ignored (the *bag of words* assumption) and we make use of the frequency of each word.

Figure from J&M 3rd ed. draft, sec 7.1

A probabilistic model that generalizes

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     $p(y' \mid \text{Filled, with, horrific}) = p(y' \mid \text{Filled, horrific, with})$

     So called because a **bag** or **multiset** is a data structure that stores counts of elements, but not their order.

A probabilistic model that generalizes

- The BoW assumption isn't enough, though, unless documents with all the same words occurred in the training data. Hence:

  2. The **naïve Bayes assumption:** Assume the words are **independent** conditioned on the class y'

     $p(\text{Filled, with, horrific} \mid y')$
     $= p(\text{Filled} \mid y') \times p(\text{with} \mid y') \times p(\text{horrific} \mid y')$

Hang on, we actually wanted:

$p(y' \mid \text{Filled, with, horrific})$

How to reverse the order?

Bayes' Rule

$$p(B \mid A) = \frac{p(B) \times p(A \mid B)}{p(A)}$$

$$p(B \mid A) = \frac{p(B) \times p(A \mid B)}{p(A)}$$

multiply both sides by p(A)

$$p(A) \times p(B \mid A) = p(B) \times p(A \mid B)$$

Chain Rule

$$p(A, B) = p(B, A)$$

...which is true by definition of joint probability

Bayes' Rule

$$p(B \mid A) = \frac{p(B) \times p(A \mid B)}{p(A)}$$

$$\underbrace{p(B \mid A)}_{\textbf{posterior}} \propto \underbrace{p(B)}_{\textbf{prior}} \times \underbrace{p(A \mid B)}_{\textbf{likelihood}}$$

A probabilistic model that generalizes

- The BoW assumption isn't enough, though, unless documents with all the same words occurred in the training data. Hence:

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    $$p(\text{Filled, with, horrific} \mid y')$$
    $$= p(\text{Filled} \mid y') \times p(\text{with} \mid y') \times p(\text{horrific} \mid y')$$

Hang on, we actually wanted:

$$p(y' \mid \text{Filled, with, horrific})$$

How to reverse the order?

A probabilistic model that generalizes

- The BoW assumption isn't enough, though, unless documents with all the same words occurred in the training data. Hence:

  2. The **naïve Bayes assumption:** Assume the words are **independent** conditioned on the class y'

$$p(\text{Filled, with, horrific} \mid y') = p(\text{Filled} \mid y') \times p(\text{with} \mid y') \times p(\text{horrific} \mid y')$$

$$p(y' \mid \text{Filled, with, horrific}) \propto p(y') \times p(\text{Filled, with, horrific} \mid y') = p(y') \times p(\text{Filled} \mid y') \times p(\text{with} \mid y') \times p(\text{horrific} \mid y')$$

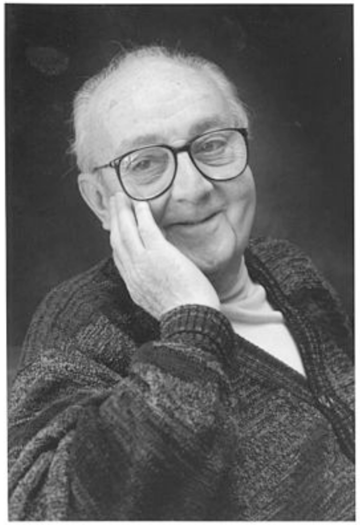
Is this a good model?

- What is wrong with these assumptions?

Is this a good model?

- George Box, statistician: “essentially, **all models are wrong**, **but** **some are useful**”)

- It turns out that naïve Bayes + BoW works pretty well for many text classification tasks, like spam detection.



Naïve Bayes Classifier

$w_j \leftarrow [\mathbf{words}(x)]_j$

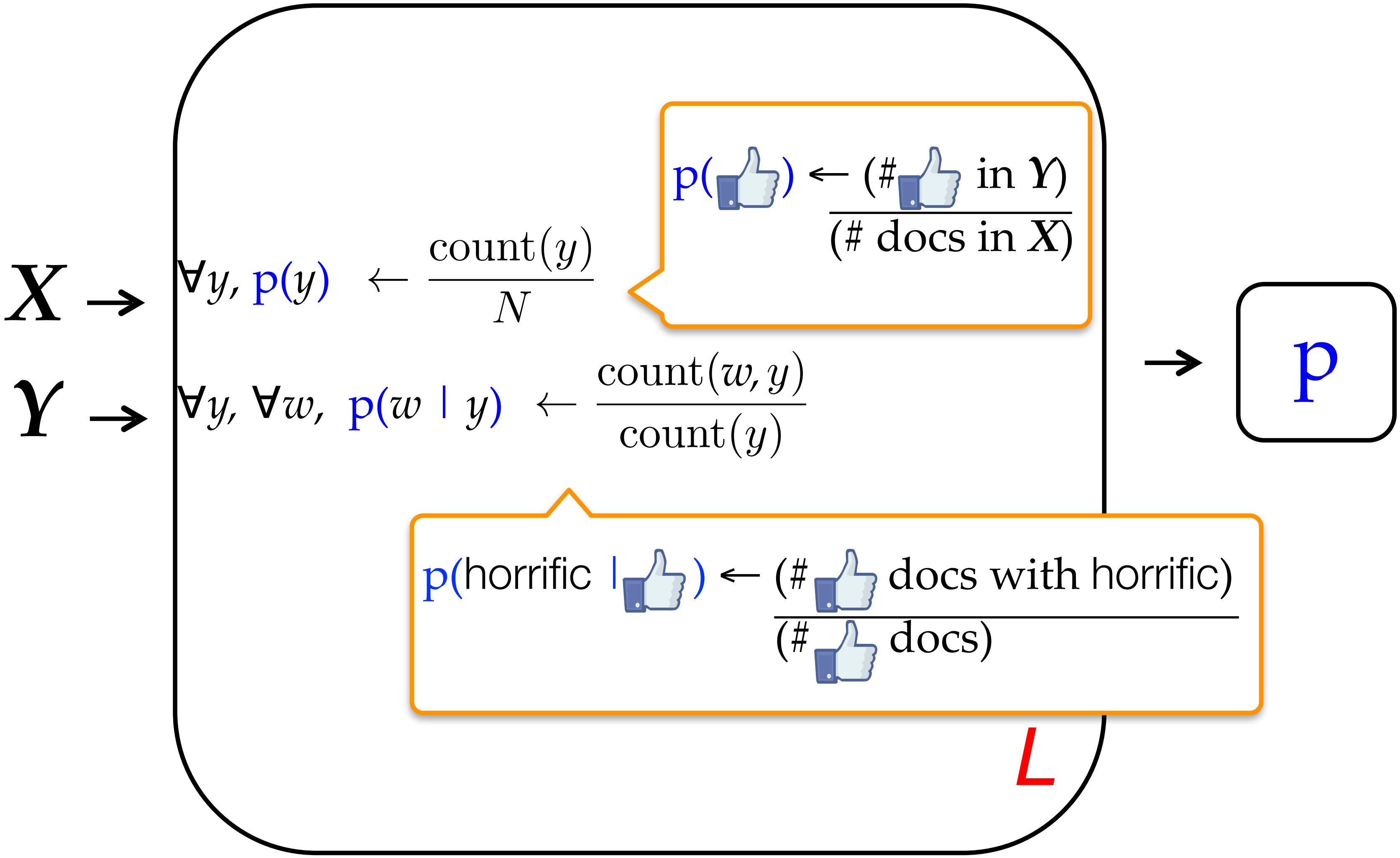
return

$\arg\max_{y'} p(y') \times \prod_j p(w_j \mid y')$

In other words: Loop over class labels, choose the one that makes the document most probable (prior × likelihood)

$x \rightarrow$ C $\rightarrow y$

Naïve Bayes Learner



Parameters

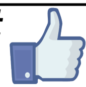
- Each probability (or other value) that is **learned** and used by the classifier is called a **parameter**
  - E.g., a single probability in a distribution
- Naïve Bayes has two kinds of distributions:
  - the class prior distribution, **p(y)**
  - the likelihood distribution, **p(w | y)**
- So how many parameters total, if there are K classes and V words in the training data?

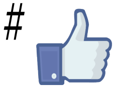
Smoothing p(*w* | *y*)

$$p(\text{horrific} \mid 👍) \leftarrow \frac{(\#\, 👍 \text{ docs with horrific})}{(\#\, 👍 \text{ docs})}$$

- What if we encounter the word distraught in a test document, but it has never been seen in training?
  - Can’t estimate p(distraught | 👍) or p(distraught | 👎): numerator will be 0
  - Because the word probabilities are multiplied together for each document, the probability of the whole document will be 0

Smoothing p(w | y)

p(horrific | ) ← [(#  docs with horrific) + 1] / [(#  docs) + V + 1]

p(OOV | ) ← 1 / [(#  docs) + V + 1]

V is the size of the vocabulary of the training corpus

- **Smoothing** techniques adjust probabilities to avoid **overfitting** to the training data
  - Above: **Laplace (add-1) smoothing**
  - OOV (out-of-vocabulary/unseen) words now have small probability, which decreases the model's confidence in the prediction without ignoring the other words
  - Probability of each seen word is reduced slightly to save probability mass for unseen words

Smoothing p(w | y)

$$p(\text{horrific} \mid 👍) \leftarrow \frac{(\#👍\ \text{docs with horrific}) + 1}{(\#👍\ \text{docs}) + V + 1}$$

$$p(\text{OOV} \mid 👍) \leftarrow \frac{1}{(\#👍\ \text{docs}) + V + 1}$$

V is the size of the vocabulary of the training corpus

- **Laplace (add-1) smoothing**, above, uses a **pseudo-count** of 1, which is kind of arbitrary.
    - For some datasets, it's overkill—better to smooth less.
    - **Lidstone (add-α) smoothing**: **tune** the amount of smoothing on **development** data:

$$p(\text{horrific} \mid 👍) \leftarrow \frac{(\#👍\ \text{docs with horrific}) + \alpha}{(\#👍\ \text{docs}) + \alpha(V + 1)}$$

$$p(\text{OOV} \mid 👍) \leftarrow \frac{\alpha}{(\#👍\ \text{docs}) + \alpha(V+1)}$$

Naïve Bayes Classifier

$x \rightarrow$

$w_j \leftarrow [\mathbf{words}(x)]_j$

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In other words: Loop over class labels, choose the one that makes the document most probable (prior × likelihood)

C

$\rightarrow y$

Can get very small

Avoiding Underflow

- Multiplying 2 very small floating point numbers can yield a number that is too small for the computer to represent. This is called **underflow**.

- In implementing probabilistic models, we use **log probabilities** to get around this.
  - Instead of storing p(•), store log p(•)
  - p(•) × p′(•) → log p(•) + log p′(•)
  - p(•) + p′(•) → `numpy.logaddexp`(log p(•), log p′(•))

Noisy Channel Model

$p(y)$ $\quad$ $p(x \mid y)$

source $\rightarrow$ y $\rightarrow$  $\rightarrow$ x

channel

decode

What proportion of emails are expected to be spam vs. not spam?

What proportion of product reviews are expected to get 1,2,3,4,5 stars?

Noisy Channel Classifiers

$x \rightarrow$

return

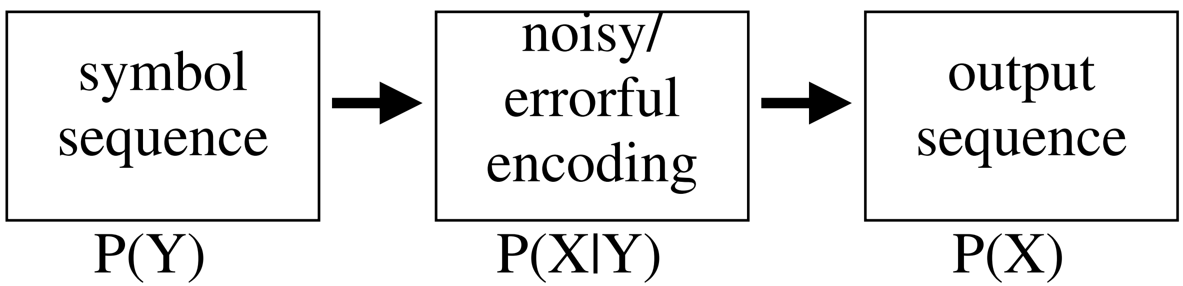
$\arg\max_y \, p(y) \times p(x \mid y)$

c

$\rightarrow y$

Noisy Channel Model

- We imagine that someone tries to communicate a sequence to us, but noise is introduced. We only see the output sequence.



Application	Y	X
Speech recognition	spoken words	acoustic signal
Machine translation	words in L_1	words in L_2
Spelling correction	intended words	typed words

NC Example: Spelling Correction

- $P(Y)$: Distribution over the words the user intended to type. A language model!

- $P(X|Y)$: Distribution describing what user is **likely** to type, given what they **meant**. Could incorporate information about common spelling errors, key positions, etc. Call it a **noise model**.

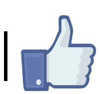
- $P(X)$: Resulting distribution over what we actually see.

- Given some particular observation x (say, effert), we want to recover the most probable y that was intended.

Conclusions

- We have seen how labeled **training data** and **supervised learning** can produce a better-informed classifier
    - Classifier takes an *input* (such as a text document) and predicts an *output* (such as a class label)
    - Learner takes *training data* and produces (statistics necessary for) the classifier

Conclusions

- Because most pieces of text are unique, it's not very practical to assume the one being classified has occurred in the training data
  - We need to make **modeling assumptions** that help the learner to **generalize** to unseen inputs
- The **naïve Bayes** model + **bag-of-words** assumption are a simple, fast probabilistic approach to text classification
  - Works well for many tasks, despite being a ~~dumb~~ naïve model of language: We know that
    - *good, not as bad as expected ≠ bad, not as good as expected*
    - p(Star Wars |) ≠ p(Star |) × p(Wars |)

Conclusions

- In practice, we need **smoothing** to avoid assuming that everything that might come up at test time is in the training data

- Implementation trick: use **log probabilities** to avoid underflow