Problem Set #2: Logic and Probability

IDST-010-06

Due: Thursday, 13 October 2016, 11:59 PM

Instructions. To solve the following problems on logic and probability, you can work on your own, seek help from me during office hours, or ask questions during class or using the discussion board. You can also consult inanimate outside sources, but you must acknowledge the use of these sources on all versions of your solution. You can not give other students answers, but you can point them in the right direction in class or on the discussion board so everyone can benefit from your advice. Show all of your work, even for solutions that seem trivial or obvious. Feel free to submit a hand-written solution. For electronic submissions, upload a PDF version to Blackboard. Please do not submit electronically by scanning or taking pictures of hand-written solutions.

1. Show that the following argument is deductively valid.

$$\begin{array}{ll} & p \vee (q \vee r) \\ & \neg r \\ \therefore & p \vee q \end{array}$$

2. Show that the following argument is deductively invalid.

$$\begin{array}{ll} & p \rightarrow q \vee \neg r \\ & q \rightarrow p \,\&\, r \\ \therefore & p \rightarrow r \end{array}$$

3. The following formulas are known as DeMorgan's Laws. Show that the two laws are logically equivalent.

   (a) $\neg p \vee \neg q \equiv \neg(p \,\&\, q)$

   (b) $\neg p \,\&\, \neg q \equiv \neg(p \vee q)$

4. Given the premises $\{s\}$, $\{r\}$, $\{\neg p, \neg q, \neg s\}$, and $\{q\}$, and the negated conclusion $\{p, \neg r\}$, conduct a proof by resolution to derive the empty set.

5. Apologies to vegetarians: "If you don't eat your meat, you can't have any pudding!" Oliver got his pudding. Prove that Oliver ate his meat.

6. Studies show that one percent of the people in the US have a particular genetic defect. When tested, 90% of the time, the test correctly detects the presence of the defect. On the other hand, roughly 9.5% of the tests administered are false positives; that is, the outcome of the test is positive but there is no defect. If the outcome of a test for a person is positive, what is the probability that he or she has the genetic defect?