

14 PROBABILISTIC REASONING

```

function ENUMERATION-ASK( $X, \mathbf{e}, bn$ ) returns a distribution over  $X$ 
  inputs:  $X$ , the query variable
            $\mathbf{e}$ , observed values for variables  $\mathbf{E}$ 
            $bn$ , a Bayes net with variables  $\{X\} \cup \mathbf{E} \cup \mathbf{Y}$  /*  $\mathbf{Y} = \text{hidden variables}$  */

   $\mathbf{Q}(X) \leftarrow$  a distribution over  $X$ , initially empty
  for each value  $x_i$  of  $X$  do
     $\mathbf{Q}(x_i) \leftarrow$  ENUMERATE-ALL( $bn.VARS, \mathbf{e}_{x_i}$ )
    where  $\mathbf{e}_{x_i}$  is  $\mathbf{e}$  extended with  $X = x_i$ 
  return NORMALIZE( $\mathbf{Q}(X)$ )

```

```

function ENUMERATE-ALL( $vars, \mathbf{e}$ ) returns a real number
  if EMPTY?( $vars$ ) then return 1.0
   $Y \leftarrow$  FIRST( $vars$ )
  if  $Y$  has value  $y$  in  $\mathbf{e}$ 
    then return  $P(y | \text{parents}(Y)) \times$  ENUMERATE-ALL( $\text{REST}(vars), \mathbf{e}$ )
    else return  $\sum_y P(y | \text{parents}(Y)) \times$  ENUMERATE-ALL( $\text{REST}(vars), \mathbf{e}_y$ )
    where  $\mathbf{e}_y$  is  $\mathbf{e}$  extended with  $Y = y$ 

```

Figure 14.9 The enumeration algorithm for answering queries on Bayesian networks.

```

function ELIMINATION-ASK( $X, \mathbf{e}, bn$ ) returns a distribution over  $X$ 
  inputs:  $X$ , the query variable
            $\mathbf{e}$ , observed values for variables  $\mathbf{E}$ 
            $bn$ , a Bayesian network specifying joint distribution  $\mathbf{P}(X_1, \dots, X_n)$ 

   $factors \leftarrow []$ 
  for each  $var$  in ORDER( $bn.VARS$ ) do
     $factors \leftarrow$  [MAKE-FACTOR( $var, \mathbf{e}$ )| $factors$ ]
    if  $var$  is a hidden variable then  $factors \leftarrow$  SUM-OUT( $var, factors$ )
  return NORMALIZE(POINTWISE-PRODUCT( $factors$ ))

```

Figure 14.10 The variable elimination algorithm for inference in Bayesian networks.