COSC 545, Spring 2020: Problem Set $\#5$

Due: Tue., 4/28, at the beginning of class (submit electronically on Canvas).
Covers: Lectures 23 and 26.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Prove that if 3SAT is in L then P = NP.

2. Show that A_{NFA} is NL-complete.

3. The hierarchy theorems provide a powerful tool for separating complexity classes. But there are limits to its use. Point out which sentence is wrong in the following argument that P $\neq$ NP, then explain why it is wrong.

   > Assume for contradiction that P $=$ NP. It follows that $SAT \in$ P. Therefore, $SAT \in TIME(n^k)$ for some k. Because we can reduce every language in NP to SAT, it follows that NP $\subseteq TIME(n^k)$. The time hierarchy theorem, however, tells us that there is a language A in $TIME(n^{k+1})$ that is not $TIME(n^k)$. It would follow that $A \in P$ but $A \notin NP$—a contradiction to our assumption $P =$ NP.

4. The space hierarchy theorem holds only for a "reasonable" (i.e., *space constructible*) function $f : \mathbb{N} \rightarrow \mathbb{N}$. Assume we did not have this restriction that f is "reasonable.". Define an unreasonable function $f : \mathbb{N} \rightarrow \mathbb{N}$ for which the space hierarchy proof would not work.

5. Provide a clear and concise explanation for why the Relativization Theorem tells us it is unlikely that we can prove P = NP by coming up with a simulator that can simulate any NTM in deterministic polynomial time.