COSC 545, Spring 2020: Problem Set #3

Due: Tue., 3/31, must be submitted electronically by beginning of class (submission instructions on web site)

Covers: Lectures 14 to 18.

Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. For a language L, let MIX(L) be the set consisting of every string w such that you can generate w by rearranging the letters of some $w' \in L$. Let L_{GU} be the language consisting of the last name of every student to ever attend Georgetown University.

Prove that the language $A_{GU} = \{w \mid w \in MIX(L_{GU})\}$ is in P.

- 2. Prove that $A_{NFA} = \{ \langle N, w \rangle \mid N \text{ is an NFA that accepts string } w \}$ is in *P*.
- 3. Explain what goes wrong with the Cook-Levin Theorem if we use a *window* of size 2×2 instead of 2×3 in the definition of ϕ_{move} .
- 4. Show that the language class NP is closed under union and concatenation operators.
- 5. Let language $MDST = \{\langle G, k \rangle \mid G \text{ is an undirected graph that contains a spanning tree with maximum degree k}. Prove that <math>MDST$ is NP-complete. In proving your response, you can assume that the following language is NP-Hard: $UHAM = \{\langle G \rangle \mid G \text{ is an undirected graph that contains a Hamiltonian path (i.e., a path that contains every vertex in G exactly once)}.$