COSC 545, Spring 2020: Problem Set #3

Due: Tue., 3/31, must be submitted electronically by beginning of class (submission instructions on web site)

Covers: Lectures 14 to 18.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. For a language $L$, let $MIX(L)$ be the set consisting of every string $w$ such that you can generate $w$ by rearranging the letters of some $w' \in L$. Let $L_{GU}$ be the language consisting of the last name of every student to ever attend Georgetown University.

   Prove that the language $A_{GU} = \{ w | w \in MIX(L_{GU}) \}$ is in $P$.

2. Prove that $A_{NFA} = \{ \langle N, w \rangle | N$ is an NFA that accepts string $w \}$ is in $P$.

3. Explain what goes wrong with the Cook-Levin Theorem if we use a window of size $2 \times 2$ instead of $2 \times 3$ in the definition of $\phi_{move}$.

4. Show that the language class NP is closed under union and concatenation operators.

5. Let language $MDST = \{ \langle G, k \rangle | G$ is an undirected graph that contains a spanning tree with maximum degree $k \}$. Prove that $MDST$ is NP-complete. In proving your response, you can assume that the following language is NP-Hard: $UHAM = \{ \langle G \rangle | G$ is an undirected graph that contains a Hamiltonian path (i.e., a path that contains every vertex in $G$ exactly once) \}. 
