## COSC 545, Spring 2019: Problem Set #3

**Due:** Thur., 3/28, at the beginning of class (hand in hard copy). **Covers:** Lectures 14 to 17.

**Collaboration:** You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

## Problems

1. For a language L, let MIX(L) be the set consisting of every string w such that you can generate w by rearranging the letters of some  $w' \in L$ . Let  $L_{GU}$  be the language consisting of the last name of every student to ever attend Georgetown University.

Prove that the language  $A_{GU} = \{w \mid w \in MIX(L_{GU})\}$  is in P.

- 2. Prove that  $A_{NFA} = \{ \langle N, w \rangle \mid N \text{ is an NFA that accepts string } w \}$  is in P.
- 3. Explain what goes wrong with the Cook-Levin Theorem if we use a *window* of size  $2 \times 2$  instead of  $2 \times 3$  in the definition of  $\phi_{move}$ .
- 4. Show that the language class NP is closed under union and concatenation operators.
- 5. Let language  $MDST = \{\langle G, k \rangle \mid G \text{ is an undirected graph that contains a spanning tree with maximum degree <math>k\}$ . Prove that MDST is NP-complete. In proving your response, you can assume that the following language is NP-Hard:  $UHAM = \{\langle G \rangle \mid G \text{ is an undirected graph that contains a Hamiltonian path (i.e., a path that contains every vertex in <math>G$  exactly once).