COSC 545, Spring 2019: Problem Set #1

Due: Thur., 2/7, at the beginning of class (hand in hard copy).
Covers: Lectures 1 to 6.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Provide formal arguments that regular languages are closed under *intersection* ($A \cap B = \{w \mid w \in A \wedge w \in B\}$) and *complement* ($\bar{A} = \{w \mid w \notin A\}$). Your arguments should use the same level of formal detail as the proof of closure under union presented in class and in the textbook.

2. In class, when proving that every regular language is described by an equivalent regular expression we quickly sketched an argument for why we can always transform a DFA into an equivalent GNFA. Make this sketch more concrete by concisely and clearly explaining how to transform a DFA into an equivalent GNFA.

3. For each of the following four languages, either prove it is non-regular (using the pumping lemma) or prove it regular (by drawing the state diagram of the finite automaton that accepts it):

   (a) $\{w \mid w \in \{0,1\}^* \setminus \{0,1,00\}\}$ (note: $\setminus$ is the *set minus* operator).

   (b) $\{w_1 w_2 w_1 \mid w_1, w_2 \in \{0,1\}^+\}$.

   (c) $\{a^i b^j \mid i, j > 0,\ i$ and j have different parities$\}$ (i.e., one is even and the other is odd).

   (d) $\{w\#y \mid w, y \in \{a,b\}^*, w \neq y\}$.
   (Note: The solution I have in mind for this final case is more involved than the solutions for the previous cases. You might find it useful to use what you proved in problem 1 about intersection and complement.)

4. Consider the following language of binary palindromes: $\{w \mid w \in \{0,1\}^*, w = w^{\mathcal{R}}\}$. (Recall that $w^{\mathcal{R}}$ is the string w backwards; e.g., if $w = 111000$ then $w^{\mathcal{R}} = 000111$.)

   (a) Give a CFG that generates this language.

   (b) Draw the state diagram for a PDA that accepts this language.