COSC 545, Spring 2017: Problem Set $\#5$

Due: Tue., 4/25, at the beginning of class (hand in hard copy).
Covers: Lectures 21 and 22.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Show that A_{NFA} is NL-complete.

2. Prove the following statement: if $\overline{PATH} \in$ NL, then it follows that coNL $\subseteq NL$.

   (Hint: for every $A \in$ NL, we know how to prove $A \leq_L PATH$. What does this tell us about $\bar{A}$ and $\overline{PATH}$?)

3. The hierarchy theorems provide a powerful tool for separating complexity classes. But there are limits to its use. Point out which sentence is wrong in the following argument that P $\neq$ NP, then explain why it is wrong.

   > Assume for contradiction that P $=$ NP. It follows that $SAT \in$ P. Therefore, $SAT \in TIME(n^k)$ for some k. Because we can reduce every language in NP to SAT, it follows that NP $\subseteq TIME(n^k)$. The time hierarchy theorem, however, tells us that there is a language A in $TIME(n^{k+1})$ that is not $TIME(n^k)$. It would follow that $A \in P$ but $A \notin NP$—a contradiction to our assumption $P =$ NP.

4. Use the time hierarchy theorem (and some algebra) to prove P $\subset$ EXPTIME.

5. Provide a clear and concise explanation for why the Relativization Theorem tells us it is unlikely that we can prove P = NP by coming up with a simulator that can simulate any NTM in deterministic polynomial time.