COSC 545, Spring 2017: Problem Set #3

Due: Tue., 3/21, at the beginning of class (hand in hard copy).
Covers: Lectures 14 to 17.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. For a language \( L \), let \( MIX(L) \) be the set consisting of every string \( w \) such that you can generate \( w \) by rearranging the letters of some \( w' \in L \). Let \( L_{GU} \) be the language consisting of the last name of every student to ever attend Georgetown University.
   
   Prove that the language \( A_{GU} = \{ w \mid w \in MIX(L_{GU}) \} \) is in \( P \).

2. Prove that \( A_{NFA} = \{ \langle N, w \rangle \mid N \text{ is an NFA that accepts string } w \} \) is in \( P \).

3. Explain what goes wrong with the Cook-Levin Theorem if we use a window of size \( 2 \times 2 \) instead of \( 2 \times 3 \) in the definition of \( \phi_{\text{move}} \).

4. Show that the language class NP is closed under union and concatenation operators.

5. Let language \( LP = \{ \langle G, a, b, k \rangle \mid G \text{ is an undirected graph that contains a simple path of length at least } k \text{ from } a \text{ to } b \} \). Prove that \( LP \) is NP-complete. In proving your response, you can assume that the \( UHAMPATH \) language (the undirected Hamiltonian path language), defined in Chapter 7 of Sipser, is NP-complete. Assume that \( \langle k \rangle \) encodes \( k \) in binary. Recall that a simple path does not repeat any vertices.