COSC 545, Spring 2016: Problem Set #5

Due: Tue., 4/26, at the beginning of class (hand in hard copy).
Covers: Lectures 21 to 23.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Show that $A_{NFA}$ is NL-complete.

2. Prove the following statement: if $PATH \in NL$, then it follows that $NL = coNL$.
   (Hint: for every $A \in NL$, we know how to prove $A \leq_L PATH$. What does this tell us about $\overline{A}$ and $PATH$?)

3. Point out what sentence is wrong in the following argument that $P \neq NP$. Then explain why it is wrong.

   Assume for contradiction that $P = NP$. It follows that $SAT \in P$. Therefore, $SAT \in TIME(n^k)$ for some $k$. Because we can reduce every language in NP to SAT, it follows that $NP \subseteq TIME(n^k)$. The time hierarchy theorem, however, tells us that there is a language $A$ in $TIME(n^{k+1})$ that is not $TIME(n^k)$. It would follow that $A \in P$ but $A \notin NP$—a contradiction to our assumption $P = NP$.

4. Use the time hierarchy theorem to help prove $P \subset EXPTIME$.

5. Provide a clear and concise explanation for why the Relativization Theorem tells us it is unlikely that we can prove $P = NP$ by coming up with a simulator that can simulate any NTM in deterministic polynomial time.