

COSC 545, Spring 2016: Problem Set #5

Due: Tue., 4/26, at the beginning of class (hand in hard copy).

Covers: Lectures 21 to 23.

Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Show that A_{NFA} is NL-complete.
2. Prove the following statement: if $\overline{PATH} \in \text{NL}$, then it follows that $\text{NL} = \text{coNL}$.
(Hint: for every $A \in \text{NL}$, we know how to prove $A \leq_L \text{PATH}$. What does this tell us about \bar{A} and \overline{PATH} ?)
3. Point out what sentence is wrong in the following argument that $\text{P} \neq \text{NP}$. Then explain why it is wrong.

Assume for contradiction that $\text{P} = \text{NP}$. It follows that $\text{SAT} \in \text{P}$. Therefore, $\text{SAT} \in \text{TIME}(n^k)$ for some k . Because we can reduce every language in NP to SAT , it follows that $\text{NP} \subseteq \text{TIME}(n^k)$. The time hierarchy theorem, however, tells us that there is a language A in $\text{TIME}(n^{k+1})$ that is not $\text{TIME}(n^k)$. It would follow that $A \in \text{P}$ but $A \notin \text{NP}$ —a contradiction to our assumption $\text{P} = \text{NP}$.

4. Use the time hierarchy theorem to help prove $\text{P} \subset \text{EXPTIME}$.
5. Provide a clear and concise explanation for why the Relativization Theorem tells us it is unlikely that we can prove $\text{P} = \text{NP}$ by coming up with a simulator that can simulate any NTM in deterministic polynomial time.