

COSC 545, Spring 2016: Problem Set #4

Due: 4/12, at the beginning of class (hand in hard copy).

Covers: Lectures 18 to 21.

Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Let $ANA = \{\langle w_1, w_2 \rangle \mid w_1 \text{ and } w_2 \text{ are anagrams}\}$. Assume I am able to prove $TQBF \leq_p ANA$. Show that this implies $PSPACE = P$.
2. This problem contains two parts that concern the language $EQ_{NFA} = \{\langle N_1, N_2 \rangle \mid N_1 \text{ and } N_2 \text{ are NFAs and } L(N_1) = L(N_2)\}$.
 - (a) Prove that $\overline{EQ_{NFA}} \in NPSPACE$.
 - (b) Use your answer from Part (a) to prove that $EQ_{NFA} \in PSPACE$.
3. In class and in Theorem 8.14 of the Sipser textbook we proved that GG (Generalized Geography) is $PSPACE$ -complete. This proof required us to construct a graph structure of the form shown in Figure 8.16. Consider the following update to this graph structure:

In the original proof, we include a column of diamonds gadgets, one for each variable x_i . (This structure is highlighted in Figure 8.15.) In more detail, each x_i has a 4-node diamond, consisting of a *top* node with directed edges to a *left* and *right* node, each with a directed edge to the *bottom* node.

In our new version of the structure, we replace each diamond with a hexagon. In more detail, we point the outgoing edge from *left* to a new node, *left2*, then point an outgoing edge from *left2* to the *bottom* node. We add a *right2* node in a similar fashion. The gadgets are otherwise connected as in the original proof, that is: the *bottom* of one gadget connects to the *top* below, and the edges incoming from the right side can stay the same (each goes to a single *left* or *right* node).

Describe where the proof of Theorem 8.14 (GG is $PSPACE$ -complete) breaks when we switch to this modified version of the graph structure.

4. Prove that $A = \{\langle \phi, w \rangle \mid \phi \text{ is a cnf (conjunctive normal form) boolean formula, } w \text{ is an assignment of values to the variables in } \phi, \text{ and this assignment causes } \phi \text{ to evaluate to true}\}$ is in L .

You can assume that the boolean variables in ϕ are encoded in the input in such a way that you can fit a constant number of them on a log-space working tape.

In your answer, be specific about how you check the format of the input string. Also be sure that your solution works for all cnf boolean formulas (and not just those in 3-cnf format).