COSC 545, Spring 2014: Problem Set #3

Due: Tue., 3/25, at the beginning of class (hand in hard copy).
Covers: Lectures 14 to 17.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. For a language L, let $MIX(L)$ be the set consisting of every string w such that you can generate w by rearranging the letters of some $w' \in L$. Let L_{GU} be the language consisting of the last name of every student to ever attend Georgetown University.

   Prove that the language $A_{GU} = \{w \mid w \in MIX(L_{GU})\}$ is in P.

2. Let language $L = \{\langle x, y, z, p\rangle \mid x, y, z, p \text{ are integers, } y \text{ is a power of 2, and } x^y \equiv z \bmod p\}$. Assume that $\langle x, y, z, p\rangle$ encodes the values in binary format. Prove that L is in P.

3. Explain what goes wrong with the Cook-Levin Theorem if we use a *window* of size 2×2 instead of 2×3 in the definition of ϕ_{move}.

4. Let language $LP = \{\langle G, a, b, k\rangle \mid G \text{ is an undirected graph that contains a simple path of length at least } k \text{ from } a \text{ to } b\}$. Prove that LP is NP-complete. In proving your response, you can assume that the $UHAMPATH$ language (the undirected Hamiltonian path language), defined in Chapter 7 of Sipser, is NP-complete. Assume that $\langle k\rangle$ encodes k in binary. Recall that a *simple* path does not repeat any vertices.