

## COSC 545, Spring 2014: Problem Set #2

**Due:** Thur., 2/20, at the beginning of class (hand in hard copy).

**Covers:** Lectures 7 to 10.

**Collaboration:** You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

### Problems

1. We say that a state  $q$  of PDA  $P$  is *useless* if no string causes  $P$  to enter state  $q$ . Consider the problem of determining whether a given PDA has any useless states. Formalize this problem as a language and prove it is decidable. In constructing your proof, you may assume that  $E_{PDA}$  (the language of PDA's with empty languages) is decidable.
2. We now consider the problem of testing whether a TM has any useless states. In the following, assume we ignore the *accept* and *reject* states when discussing useless states. (That is, a TM has a useless state if there is some state, other than the accept or reject states, that it never enters.)
  - (a) Given an arbitrary TM  $M$ , describe how to modify  $M$  into a machine  $M'$  such that  $L(M) = L(M')$ , and  $M'$  does not have any useless states.
  - (b) Formalize the problem of testing a TM for useless states as a language, and then prove it is undecidable using a reduction argument. You might find it useful to borrow ideas from your answer to part (a).
  - (c) Explain why we cannot apply Rice's Theorem to prove the language from part (b) is undecidable.
3. Prove that  $\overline{HALT_{TM}}$  is not Turing Recognizable.