COSC 545, Spring 2014: Problem Set #2

Due: Thur., 2/20, at the beginning of class (hand in hard copy).

Covers: Lectures 7 to 10.

Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. We say that a state $q$ of PDA $P$ is useless if no string causes $P$ to enter state $q$. Consider the problem of determining whether a given PDA has any useless states. Formalize this problem as a language and prove it is decidable. In constructing your proof, you may assume that $E_{PDA}$ (the language of PDA’s with empty languages) is decidable.

2. We now consider the problem of testing whether a TM has any useless states. In the following, assume we ignore the accept and reject states when discussing useless states. (That is, a TM has a useless state if there is some state, other than the accept or reject states, that it never enters.)

   (a) Given an arbitrary TM $M$, describe how to modify $M$ into a machine $M’$ such that $L(M) = L(M’)$, and $M’$ does not have any useless states.

   (b) Formalize the problem of testing a TM for useless states as a language, and then prove it is undecidable using a reduction argument. You might find it useful to borrow ideas from your answer to part (a).

   (c) Explain why we cannot apply Rice’s Theorem to prove the language from part (b) is undecidable.

3. Prove that $\overline{HALT_{TM}}$ is not Turing Recognizable.