COSC 545, Spring 2013: Problem Set $\#6$

Due: Tue., 4/23, at the beginning of class (hand in hard copy).
Covers: Lectures 21 to 25.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Show that A_{NFA} is NL-complete.

2. Point out what sentence is wrong in the following argument that P $\neq$ NP. Then explain why it is wrong.

   > Assume for contradiction that P $=$ NP. It follows that $SAT \in$ P. Therefore, $SAT \in TIME(n^k)$ for some k. Because we can reduce every language in NP to SAT, it follows that NP $\subseteq TIME(n^k)$. The time hierarchy theorem, however, tells us that there is a language L in $TIME(n^{k+1})$ that is not $TIME(n^k)$. It would follow that $L \in P$ but $L \notin NP$—a contradiction to our assumption $P =$ NP.

3. Our goal with this problem is to prove that there exists an oracle A such that NP$^A \neq$ coNP^A. The problem proceeds in two parts.

   (a) For a given A, let $L_A = \{x \mid \forall y, |y| = |x| : y \notin A\}$. Prove that for every A: $L_A \in$ coNP^A.

   (b) Construct a specific A such that $L_A \notin$ NP^A. (To do so, modify the relevant parts of the proof argument of Theorem 9.20 in Sipser.)

4. In class, we studied a *minimum vertex cover* algorithm that guarantees a 2-approximation. Consider the following alternative algorithm:

   > Choose the node in the graph with the largest degree. Add this node to the vertex cover and remove it and its neighbors from the graph. Repeat on the remaining graph until no more nodes remain.

   Prove that this algorithm guarantees only an $\Omega(n)$-approximation.