COSC 545, Spring 2013: Problem Set $\#5$

Due: Tue., 4/9, at the beginning of class (hand in hard copy).
Covers: Lectures 18 to 21.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Prove the following: $TQBF \leq_p 3SAT \Rightarrow PSPACE = NP$.

2. Let $EQ_{NFA} = \{\langle N_1, N_2 \mid N_1$ and N_2 are NFAs and $L(N_1) = L(N_2)\}$. Prove that $EQ_{NFA} \in PSPACE$.

3. In class and in Theorem 8.14 of Sipser we proved that GG (Generalized Geography) is $PSPACE$-complete. This proof required us to construct a graph structure of the form shown in Figure 8.16. Consider the following update to this graph structure:

   > In the original proof, we include a column of diamonds gadgets, one for each variable x_i. (This structure is highlighted in Figure 8.15.) In more detail, each x_i has a 4-node diamond, consisting of a *top* node with directed edges to a *left* and *right* node, each with a directed edge to the *bottom* node.
   >
   > In our new version of the structure, we replace each diamond with a hexagon. In more detail, we point the outgoing edge from *left* to a new node, *left2*, then point an outgoing edge from *left2* to the *bottom* node. We add a *right2* node in a similar fashion. The gadgets are otherwise connected as in the original proof, that is: the *bottom* of one gadget connects to the *top* below, and the edges incoming from the right side can stay the same (each goes to a single *left* or *right* node).

   Describe where the proof of Theorem 8.14 (GG is $PSPACE$-complete) breaks when we switch to this modified version of the graph structure.

4. We can represent a linked list by a directed graph in which each node has exactly one out-going edge. We call a linked list *proper* if traversing the edges visits every node in the graph. Let $LL = \{\langle D \rangle \mid D$ is a proper lined list $\}$. Prove that $LL \in L$.