COSC 545, Spring 2013: Problem Set $\#4$

Due: Tue., 3/26, at the beginning of class (hand in hard copy).
Covers: Lectures 13 to 17.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. We define the *bounded language* of an NFA N, denoted $L^B(N)$, to be the set of all strings that are accepted by N on a branch that never visits a state more than once. Fix some NFA N. Prove that the language $A_N = \{w \mid w \in L^B(N)\}$ is in P.

2. Let language $L = \{\langle x, y, z, p \rangle \mid x, y, z, p$ are integers, y is a power of 2, and $x^y \equiv z \bmod p\}$. Assume that $\langle x, y, z, p \rangle$ encodes the values in binary format. Prove that L is in P.

3. Let language $SP = \{\langle G, a, b, k \rangle \mid G$ is an undirected graph that contains a path of length at most k from a to $b\}$. Prove that $SP \in P$. You can assume that k is encoded in binary.

4. Prove the following: $SAT \in P \Rightarrow L$ is NP-complete (where L is the language from problem 2).

5. Let language $LP = \{\langle G, a, b, k \rangle \mid G$ is an undirected graph that contains a simple path of length at least k from a to $b\}$. Prove that LP is NP-complete. In proving your response, you can assume that the $UHAMPATH$ language (the undirected Hamiltonian path language), defined in Chapter 7 of Sipser, is NP-complete. As before, you can assume that k is encoded in binary. Recall that a *simple* path does not repeat any vertices.