COSC 545, Spring 2013: Problem Set #2

Due: Tue., 2/12, at the beginning of class (hand in hard copy).
Covers: Lectures 4 to 6.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. The following three question parts concern the argument from Chapter 2 that push down automata and context-free grammars are equivalent.

   (a) Draw a state machine diagram for a PDA that accepts the same language as this CFG:

   $S \to S_1$
   $S_1 \to 0S_11 \mid 11$

   (b) Lemma 2.27 in the book argues that if a PDA accepts some language, then there is a CFG that generates the same language. The proof starts fixing some PDA P then constructing a CFG G which it then argues (in Claims 2.30 and 2.31) generates the same language as the one accepted by P. Let G' be defined the same as G except we now add the additional set of rules:

   $\forall p \in Q, \forall a \in \Sigma$: $A_{pp} \to a$

   Identify a specific place in the proof that breaks when we use G' instead of G. Explain why it breaks.

   (c) Let an *observant pushdown automaton* (OPDA) be defined the same as a PDA with the exception that it now has a *peek* operation that allows it peek at the top of the stack without pushing or popping. In more detail, a peek transition, written $a \to$, is followed if the top of the stack contains symbol a. The transition does not modify the stack. For example, $\mathbf{q_1}\ \ b \to\ \ \mathbf{q_2}$ means: *if the machine is in state q_1 and the symbol at the top of the stack is b, then transition to q_2 without altering the stack.*

   Prove that a language is context free if and only if some OPDA recognizes it. (You can use any theorem proved in class.)

2. Use the pumping lemma for context-free languages to prove that following language is not context-free: $\{ww^Rw \mid w \in \{0,1\}^*\}$. (Note: w^R describes the string w written in reverse order. For example, for $w = 110$, $w^R = 011$.)

3. Prove that the following language is Turing decidable: $\{\langle R, S\rangle \mid R$ and S are regular expressions and $L(R) \subseteq L(S)\}$. In proving this result you may use any result proved in Chapter 4 or earlier, as well as any result proved in problem set 1.