COSC 545, Spring 2013: Problem Set $\#1$

Due: Tue., 1/29, at the beginning of class (hand in hard copy).
Covers: Lectures 1 to 4.
Collaboration: You must work alone on the problem set and not consult outside sources. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Provide formal arguments that regular languages are closed under *intersection* ($A \cap B = \{w \mid w \in A \wedge w \in B\}$) and *complement* ($\bar{A} = \{w \mid w \notin A\}$). Your arguments should use the same level of formal detail as the proof of closure under union presented in the textbook.

2. For each of the following three languages, either prove it is non-regular (using the pumping lemma) or prove it regular (by drawing the state diagram of the finite automaton that accepts it):

   (a) $\{w \mid w \in \{0,1\}^* \setminus \{0, 1, 00\}\}$ (note: $\setminus$ is the *set minus* operator).

   (b) $\{w_1 w_2 w_1 \mid w_1, w_2 \in \{0,1\}^*\}$.

   (c) $\{a^i b^j \mid i, j > 0,$ i and j have different parities$\}$ (i.e., one is even and the other is odd).

3. This question concerns the language $A = \{x\#y \mid x, y \in \{0,1\}^*, x \neq y\}$

   (a) Explain the mistake in the following argument that A is non-regular:

      $\bar{A} = \{x\#y \mid x, y \in \{0,1\}^*, x = y\}$. *Because regular languages are closed under negation, if A was regular then $\bar{A}$ is regular. However, a simple application of the pumping lemma (shown in the book and in class) proves that $\{x\#y \mid x, y \in \{0,1\}^*, x = y\}$ is not regular. Therefore A cannot be regular.*

   (b) Fix the argument above (*Hint:* keep the same general argument, but now also use the intersection operation to overcome the mistake from above).

4. Give a CFG generating the language of binary strings with twice as many 0's as 1's.