COSC 545, Spring 2012: Problem Set $\#5$

Due: Wed., 4/11, at the beginning of class (hand in hard copy).
Covers: Lectures 18 to 21.
Collaboration: You may collaborate with classmates. Every student must write up his or her own answers and list collaborators. No sources outside of the assigned textbook may be consulted.

Note: I am trying the same basic format as the last problem set: 3 problems of easy to medium difficulty, and one *harder* problem (in this case, problem 4; though problem 2 also has some trickiness surrounding it).

Problems

1. ***Fun with Complexity Class Definitions:*** Prove the following: if every NP-hard problem is also $PSPACE$-hard, then $PSPACE = NP$.

2. ***Memory-Bounded Turing Machines:*** Problem 8.8 from Sipser. (*Hint:* You might find useful the discussion of $\bar{A}LL_{NFA}$ from Example 8.4.)

3. ***Generalized Geography:*** In class and in Theorem 8.14 of Sipser we proved that GG (Generalized Geography) is $PSPACE$-complete. This proof required us to construct a graph structure of the form shown in Figure 8.16. Consider the following update to this graph structure:

   > In the original proof, we include a column of diamonds gadgets, one for each variable x_i. (This structure is highlighted in Figure 8.15.) In more detail, each x_i has a 4-node diamond, consisting of a *top* node with directed edges to a *left* and *right* node, each with a directed edge to the *bottom* node.
   >
   > In our new version of the structure, we replace each diamond with a hexagon. In more detail, we point the outgoing edge from *left* to a new node, *left2*, then point an outgoing edge from *left2* to the *bottom* node. We add a *right2* node in a similar fashion. The gadgets are otherwise connected as in the original proof, that is: the *bottom* of one gadget connects to the *top* below, and the edges incoming from the right side can stay the same (each goes to a single *left* or *right* node).

   Describe where the proof of Theorem 8.14 breaks when we switch to this modified version of the graph structure.

4. ***Log-Space:*** Consider the language A of properly-nested parentheses and brackets. For example: “([()][])” is properly nested, while “([)]” is not. Prove that $A \in L$.