COSC 545, Spring 2012: Problem Set #4

Due: Wed., 3/28, at the beginning of class (hand in hard copy).
Covers: Lectures 14 to 17.
Collaboration: You may collaborate with classmates. Every student must write up his or her own answers and list collaborators. No sources outside of the assigned textbook may be consulted.

Note: To keep things fresh, I am trying something different with this problem set. Instead of offering 6 − 7 problems of easy to medium difficulty, I am instead offering 3 problems of easy to medium difficulty (problems 1 − 3) and one problem that I believe to be of hard difficulty (problem 4).

Problems

1. The P Complexity Class: In the following problem, let \( \text{mix}(S) \), for set \( S \) of symbols, be the set consisting of every string made up of symbols from \( S \), such that no symbol appears more than once in the string. Fix some \( \text{CFG} \) \( G = (V, \Sigma, R, S) \). Prove that the language \( A_G = \{ S \subseteq \Sigma, \forall s \in \text{mix}(S) : s \in L(G) \} \) is in \( P \).

2. NP-Completeness: Problem 7.17 from Sipser.

   (Note: The definitions of languages \( \text{PATH} \) and \( \text{UHAMPATH} \) can be found in Chapter 7 of Sipser; also, a simple path in a graph is a path in which no node is repeated.)

4. Tricky NP-Completeness: We say an assignment to a 3cnf-formula is balanced if the assignment satisfies the formula and at least one literal in each clause evaluates to 0. Let \( BSAT \) be the collection of 3cnf-formulas that have a balanced assignment. Prove that \( BSAT \) is NP-complete.