COSC 545, Spring 2012: Problem Set #3

Due: Wed., 3/14, at the beginning of class (hand in hard copy).
Covers: Lectures 9 to 13.
Collaboration: You may collaborate with classmates. Every student must write up his or her own answers and list collaborators. No sources outside of the assigned textbook may be consulted.

A Note on TM Description Formality: When describing Turing Machines, please use the same level of detail requested for problem set 2.

Problems

1. The Post-Correspondence Problem: In class, we proved that the post-correspondence problem (PCP) is undecidable. In more detail, what we really proved is that PCP is undecidable when defined over an alphabet that can easily record a TM computation history (e.g., the alphabet had a symbol for each possible TM state, plus every possible TM input and tape symbol, and the configuration delimiter #). Here we ask what happens when we restrict the alphabet. (Be detailed in your answers; high-level intuition alone will not receive full credit.)
   (a) Show that PCP is decidable when the dominoes can only contain symbols from the alphabet \( \Sigma = \{1\} \).
   (b) Show that PCP is not decidable when the dominoes can only contain symbols from the alphabet \( \Sigma = \{0, 1\} \).

2. Computation History Method: Problem 5.34 from Sipser.

3. Bounded Turing Machines: Problem 5.27 from Sipser described a computational model called a two-dimensional finite automaton (2DIM-DFA), Let a triangular two-dimensional finite automaton (T2DIM-DFA) be defined the same but with the following exception: it automatically rejects in the first step any input rectangle with a non-blank symbol in any position \((i, j), i \leq m, j \leq n, j > i\). The following two questions ask you about this model.
   (a) Consider the problem of determining if a given T2DIM-DFA \( D \) accepts a given input rectangle \( r \). Formulate this problem as a language and then prove one of the following two statements: this language is decidable or this language is undecidable.
   (b) Consider the problem of determining whether two T2DIM-DFA machines are equivalent. Formulate this problem as a language and then prove one of the following two statements: this language is decidable or this language is undecidable.

4. Recursion Theorem: Describe two different TMs \( A \) and \( B \) such that when started on any input, \( A \) outputs \( \langle B \rangle \) and \( B \) outputs \( \langle A \rangle \). (You might find it useful to use the Recursion Theorem in these constructions.)

5. Time Complexity: Let language \( L = \{ \langle x, y, z, p \rangle \mid x, y, z, p \text{ are integers, } y \text{ is a power of 2, and } x^y \equiv z \mod p \} \). Assume that \( \langle x, y, z, p \rangle \) encodes the values in binary format. Prove that \( L \) is decidable in polynomial time. (Formally, show \( L \in TIME(f(n)) \) for some polynomial \( f \).)