COSC 545, Spring 2012: Problem Set $\#3$

Due: Wed., 3/14, at the beginning of class (hand in hard copy).
Covers: Lectures 9 to 13.
Collaboration: You may collaborate with classmates. Every student must write up his or her own answers and list collaborators. No sources outside of the assigned textbook may be consulted.

A Note on TM Description Formality: When describing Turing Machines, please use the same level of detail requested for problem set 2.

Problems

1. ***The Post-Correspondence Problem:*** In class, we proved that the post-correspondence problem (PCP) is undecidable. In more detail, what we really proved is that PCP is undecidable when defined over an alphabet that can easily record a TM computation history (e.g., the alphabet had a symbol for each possible TM state, plus every possible TM input and tape symbol, and the configuration delimiter $\#$). Here we ask what happens when we restrict the alphabet. (Be detailed in your answers; high-level intuition alone will not receive full credit.)

   (a) Show that PCP *is* decidable when the dominoes can only contain symbols from the alphabet $\Sigma = \{1\}$.

   (b) Show that PCP *is not* decidable when the dominoes can only contain symbols from the alphabet $\Sigma = \{0, 1\}$.

2. ***Computation History Method:*** Problem 5.34 from Sipser.

3. ***Bounded Turing Machines:*** Problem 5.27 from Sipser described a computational model called a *two-dimensional finite automaton* (2DIM-DFA), Let a *triangular two-dimensional finite automaton* (T2DIM-DFA) be defined the same but with the following exception: it automatically rejects in the first step any input rectangle with a non-blank symbol in any position (i, j), $i \leq m$, $j \leq n$, $j > i$. The following two questions ask you about this model.

   (a) Consider the problem of determining if a given T2DIM-DFA D accepts a given input rectangle r. Formulate this problem as a language and then prove one of the following two statements: this language is decidable *or* this language is undecidable.

   (b) Consider the problem of determining whether two T2DIM-DFA machines are equivalent. Formulate this problem as a language and then prove one of the following two statements: this language is decidable *or* this language is undecidable.

4. ***Recursion Theorem:*** Describe two different TMs A and B such that when started on *any* input, A outputs $\langle B \rangle$ and B outputs $\langle A \rangle$. (You might find it useful to use the Recursion Theorem in these constructions.)

5. ***Time Complexity:*** Let language $L = \{\langle x, y, z, p \rangle \mid x, y, z, p$ are integers, y is a power of 2, and $x^y \equiv z \bmod p\}$. Assume that $\langle x, y, z, p \rangle$ encodes the values in binary format. Prove that L is *decidable in polynomial time*. (Formally, show $L \in TIME(f(n))$ for some polynomial f.)