COSC 240, Fall 2017: Problem Set $\#1$

Assigned: Wed, 9/13/17.
Due: Wed, 9/20, at the beginning of class (hand in hard copy).
Lectures Covered: Weeks 2 to 4.
Academic Integrity: You can work with other people in the class but you must write up your own answers in your own words. You can also use the textbook and talk to the professor. You may not use any other resources (e.g., material found online) or talk to people outside the class about these problems. See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Prove the following statements using the formal definitions of asymptotic notation we learned in class (i.e., actually identify positive constants c and k for which the stated claims hold):

   (a) $5\sqrt{n} = O(n)$
   (b) $(1/2)n^2 = \Omega(n^2 + 10n + 8)$
   (c) $n^3 + n^2 + n = \Theta(n^3)$

2. Draw a recursion tree for the following recurrence (you can assume that n is a power of 3), and label it with the following: (a) the number of levels; (b) number of nodes per level; (c) cost per non-leaf level; and (d) cost of leaf level.

$$T(n) = \begin{cases} 2T(n/3) + n & \text{if } n \geq 3 \\ 1 & \text{if } n < 3 \end{cases}$$

3. Using your answers from the previous problem, find a big-Θ bound for the above recurrence and argue why it is correct. You may find it useful in your argument to use the infinite decreasing geometric series bound, which says that for $0 < r < 1$: $\sum_{i=0}^{\infty} r^k \leq \frac{1}{1-r}$.

   (Recall: to show $T(n) = \Theta(f(n))$, for some $f(n)$, you must show both that $T(n) = O(f(n))$ *and* that $T(n) = \Omega(f(n))$.)

4. Now use the master theorem to bound the recurrence $T(n) = 2T(n/3) + n$.

   (If you end up applying Case 3, make sure you also show the regularity condition; i.e., that there exists a constant $c < 1$ such that $af(n/b) \leq cf(n)$.)

5. Calculate a big-Θ bound on the *expected* step complexity of the following (weird) randomized algorithm (assume that input A is an array of length $n > 0$):

   Rand-Reset(A)
   $n \leftarrow length(A)$
   $i \leftarrow$ a random value chosen uniformly from $\{1, 2, ..., n\}$
   while $i > 0$
   $A[i] \leftarrow 0$
   $i \leftarrow i - 1$