COSC 030, Fall 2019: Problem Set #8

Assigned: Tuesday, 11/19.
Due: Tuesday, 12/3
Lectures Covered: Week 13 and Week 14 (Chapters 10.2 to 10.6, 11.1 to 11.6, 12.1 to 12.4).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Draw a graph G with 5 vertices such that $\chi(G) = 4$.
2. Prove that every graph that is not complete has a vertex cut.
3. Prove that every bipartite graph with $n > 1$ vertices has a vertex cut of size $\leq n/2$.
4. Prove that $\lambda(G) \leq n - 1$ for every simple graph G with $n > 1$ nodes.
5. Draw a graph G_1 such that $\Delta(G_1) = 4$ and $\chi(G_1) = 5$ and a graph G_2 such that $\Delta(G_2) = 4$ and $\chi(G_2) = 2$.
6. In class we proved that in an m-ary tree of height h and ℓ leaves, $h \geq \lceil \log_m(\ell) \rceil$. Prove that for any $m \geq 2$ and $\ell \geq 2$ there exists a tree with $h \geq \ell$.
7. How many edges are in a forest of n nodes and k components? Prove your answer is correct. (In your proof you may use, without reproving, any theorem we addressed in class.)
8. Prove that the boolean algebra expression $AB+C(B+BC)$ is equivalent to the expression $B(A+C)$. (You can use the identities discussed in class, but you must name any identities you use where you use them.)
9. Let $F(x, y, z)$ be the boolean function that evaluates to 1 only when exactly two of its three variables are true. Define $F(x, y, z)$ as a boolean algebra expression.
10. Draw a circuit diagram that implements the boolean function from the previous problem.