COSC 030, Fall 2019: Problem Set $\#5$

Assigned: Tuesday, 10/8.
Due: Thursday, 10/17, at the beginning of class (hand in hard copy).
Lectures Covered: Weeks 7 to 8; material on recursion and counting (Chapters 5 and 6).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Provide a recursive definition for the function $f(n) = n^2$.

   (Hint: You might find it useful to remember that $(n-1)^2 = n^2 - 2n + 1$.)

2. Provide a recursive definition for the set containing all positive powers of two (i.e., $\{2^i \mid i \in \mathbb{N}\}$). Remember to include $2^0 = 1$.

3. In class we studied a set S recursively defined as follows:

   - *Basis:* $3 \in S$.
   - *Recursive Step:* If $x \in S$ and $y \in S$ then $x + y \in S$.

   Use *structural induction* to prove that every $x \in S$ is a multiple of 3.

4. Assume a classroom has 30 students. The professor wants to choose a group of 3 students to grade a problem set. How many possible grading groups can he choose?

5. Assume we want to form a police line-up. The line-up room has 5 positions, labelled 1 to 5. We have a pool of 10 people to use in forming our 5-person line-up. One person in the pool is the actual suspect. For a line-up to be valid it must contain the suspect and the suspect must be in position 3.

   How many different valid 5-person line-ups can we form from this pool?

6. The National Park Service's employee intranet requires a password that is either five (lower case) letters long or the name of a state in the United States (in lower case). For state names with two words (e.g., New Jersey) the password must skip the space (e.g., newjersey).

   How many possible passwords are there?

   (Show your work and mention which rules you are applying to derive your answer.)

7. Consider strings of 8 bits (i.e., computer bytes). How big are the following two sets of bytes?

   - $S_1 = \{x \mid (x \text{ is byte that starts with } 1) \vee (x \text{ is a byte that ends with } 11)\}$
   - $S_2 = \{x \mid x \text{ has at least one } 0 \text{ in its last two positions } \}$

8. Assume you have a network router that stores each incoming packet in one of 3 queues that can each hold 12 packets. Assume at some point all 3 queues are empty when suddenly a group of n packets arrives all at once for the router to divide into its queues. You do not know in advance the strategy the router will use to decide in which queue to place each packet. What is the smallest value of n that guarantees at least one queue will be full (i.e., contain 12 packets)?