COSC 030, Fall 2016: Problem Set $\#7$

Assigned: Tuesday, 11/15.
Due: Tuesday, 11/22.
Lectures Covered: Week 12 and Master Theorem Material from Week 11 (Chapters 8.3, 10.1, 10.2).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Using the Master Theorem: Solve the recurrence $f(n) = f(n/2) + (n^2/2)$ (for values of n that can be expressed as a power of 2). Your answer can be in asymptotic notation.

2. Using the Master Theorem: Solve the recurrence $f(n) = 2f(n/4) + \sqrt{n}$ (for values of n that can be expressed as powers of 4). Your answer can be in asymptotic notation.

3. Draw each of the following graphs:

   - K_4
   - $K_{9,2}$
   - W_3
   - Q_2

4. Let S be the set containing every undirected pseudograph with 3 edges and the vertex set $V = \{a, b, c, d\}$. What is the size of S?

5. Let S be the set containing every simple undirected graph defined over the vertex set $V = \{a, b, c, d, e\}$.

   (a) What is the size of S?

   (b) How many graphs in S satisfy the property that vertex a has no neighbors?

6. A *vertex cover* of an undirected graph $G = (V, E)$ is a subset of vertices $C \subseteq V$ such that every edge in E contains at least one endpoint in C. In 1931, König proved that in a bipartite graph, the size of the largest matching is equal to the size of the smallest vertex cover.

   Prove the following weaker relationship between matchings and vertex covers (which is used as a step in the full proof of König's Theorem): *for any undirected simple graph $G = (V, E)$, and a vertex cover C and matching M (defined for G), the size of M is no more than the size of C.*