COSC 030, Fall 2015: Problem Set $\#6$

Assigned: Tuesday, 11/1.
Due: Tuesday, 11/15.
Lectures Covered: Weeks 10 and 11 (Chapters 7.1, 7.2, 7.4, 8.1 and 8.3).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

Note #1: When asked to provide a probability you can express the probability as a fraction (e.g., you can say, $9/115$ instead of 0.07826086956).

Note #2: For all problems you must **show your work** to get full credit. For example, describe the sample space and event definitions used in probability calculations.

Note #3: This problem set has 9 problems that span two pages. Make sure to answer the problems on the second page.

1. Assume there are three people of three different heights that are assigned with uniform randomness to three seats (i.e., each possible assignment of people to seats is equally likely). What is the probability that the tallest person ends up in the middle seat?

2. Consider a game where I roll two dice and you win if their sum is greater than 9.

   - What is the probability that you win?
   - Assume on one of the two die I replace the 6 with another 3. This die now has two sides marked 3 and no side marked 6. The other die is not changed. Now what is the probability that you win?

3. Consider a game where I roll six dice and you win if no more than four of the dice roll a one. What is the probability that you win? (You might find it useful to leverage the fact proved in class that $p(E) = 1 - P(\bar{E})$.)

4. Assume a game show where you are presented with three closed doors. There is a prize behind one of the doors. You have two choices: (a) open door 1; or (b) open doors 2 and 3. If the prize is behind a door you open, you win the prize. Assume the probability that the prize is behind door 1 is twice as large as the probability that the prize is behind door 2, and the probability that the prize is behind door 2 is twice as large as the probability that the prize is behind door 3.

   - What is the probability that you win the prize if you choose option a from above?
   - What is the probability that you win the prize if you choose option b?

5. I generate a string of 5 bits by flipping a fair coin 5 times (heads = 0 and tails = 1). Given that the first two flips are heads, what is the probability that I end up with a string containing at least three 0's in a row? (To receive full credit you must use conditional probability in calculating your answer.)

6. Consider a gambling game where you roll two dice. If both dice roll the same value, you win $\$10$. Otherwise, you lose $\$1$. What is your expected winnings for this game?

7. In class we considered the *Search For* 1 problem in which you are provided a sequence containing the integers from 1 to n in some order. To solve the problem, you must output the position in the sequence that contains 1. For example, if $n = 4$ and you are provided input $(2, 1, 3, 4)$, the correct response is 2, as the 1 is in the second position of the input sequence.

   Consider the simple algorithm that iterates through the sequence, checking each position in order until it finds the 1—at which point it returns that position. Describe a probability distribution over the possible inputs for a given input size n, such that it yields an *average case* step complexity for the above algorithm that is in $\Theta(\sqrt{n})$. Your distribution must satisfy the property that it assigns a probability of 0 to any input where the 1 is not in the first or last position.

   (*For full credit:* show the calculation of average case complexity that yields your result.)

8. Using the Master Theorem: Solve the recurrence $f(n) = f(n/2) + (1/2)n^2$ (for values of n that can be expressed as a power of 2). Your answer can be in asymptotic notation.

9. Using the Master Theorem: Solve the recurrence $f(n) = 2f(n/4) + \sqrt{n}$ (for values of n that can be expressed as powers of 4). Your answer can be in asymptotic notation.