# COSC 030, Fall 2016: Problem Set #5

**Assigned:** Tuesday, 10/11.
**Due:** Thursday, 10/20, at the beginning of class (hand in hard copy).
**Lectures Covered:** Weeks 6 to 8; material on recursion and counting (Chapters 5 and 6).
**Academic Integrity:** You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

## Problems

1. Provide a recursive definition for the function $f(n) = n^2$.

   (Hint: You might find it useful to remember that $(n-1)^2 = n^2 - 2n + 1$.)

2. Provide a recursive definition for the set containing all positive powers of two (i.e., $\{2^i \mid i \in \mathbb{N}\}$). Remember to include $2^0 = 1$.

3. In class we studied a set $S$ recursively defined as follows:

   - *Basis:* $3 \in S$.
   - *Recursive Step:* If $x \in S$ and $y \in S$ then $x + y \in S$.

   Use *structural induction* to prove that every $x \in S$ is a multiple of 3.

4. Using your answer from problem 1, describe a recursive algorithm that takes a natural number as input and then returns its square.

5. Assume a classroom has 30 students. The professor wants to choose a group of 3 students to grade a problem set. How many possible grading groups can he choose?

6. Assume we want to form a police line-up. The line-up room has 5 positions, labelled 1 to 5. We have a pool of 10 people to use in forming our 5-person line-up. One person in the pool is the actual suspect. For a line-up to be valid it must contain the suspect and the suspect must be in position 3.

   How many different valid 5-person line-ups can we form from this pool?

7. **Extra Credit:** A group of six people walk into an art museum. Assume that if any two people from this group strike up a conversation they can determine whether or not they have the same taste in art. Prove that there must exist a subset of three people from this group such that either: (a) all pairs in this subset have the same taste in art; or (b) none of the pairs in this subset have the same taste in art. Use the pigeon hole principle in formulating your answer.

   (If you come up with a real simple answer, then make sure you understand what the problem is asking—a correct answer should require several steps of reasoning.)